森林管理最適化モデルのサンゴ管理への応用

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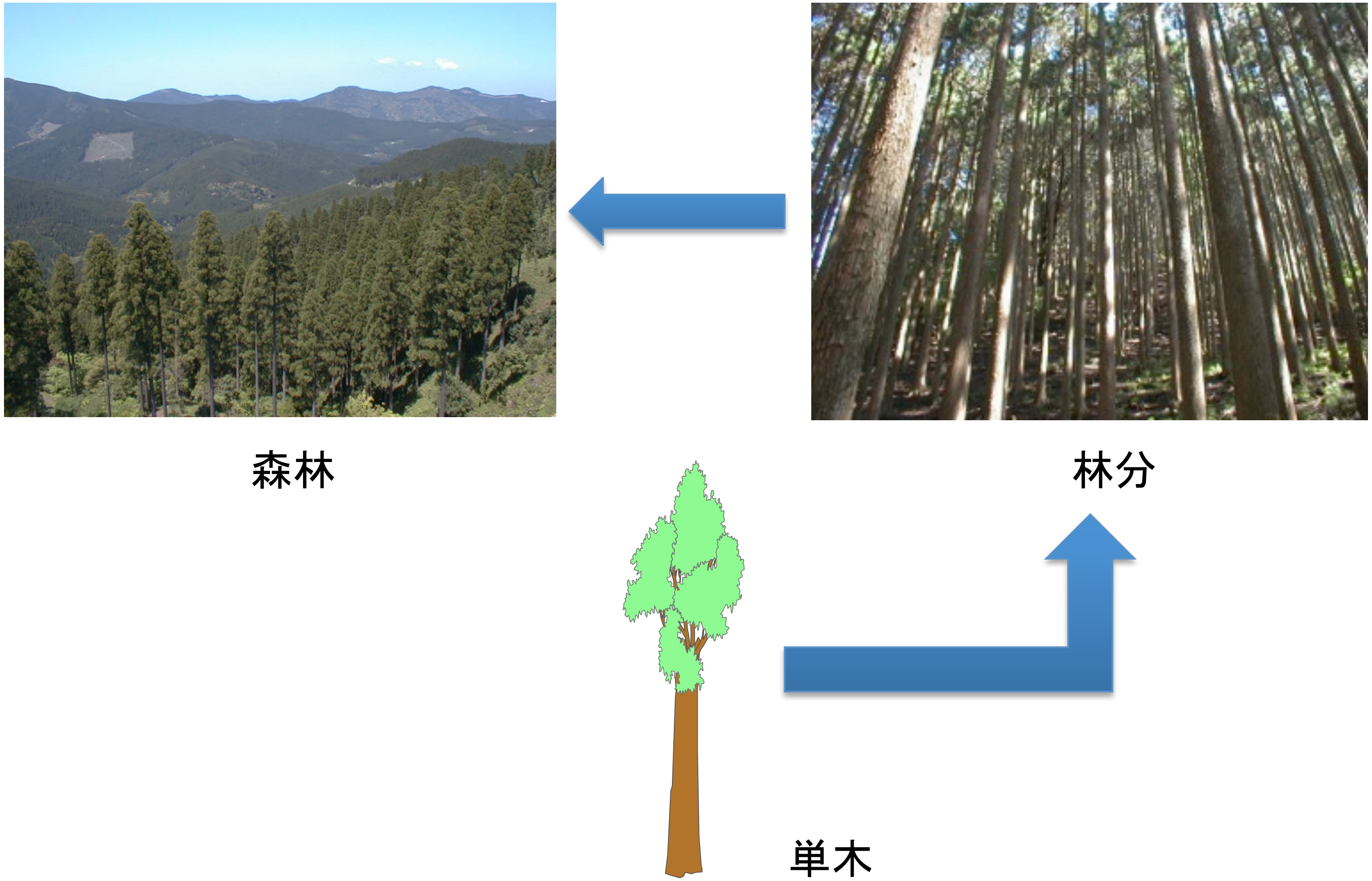
令和6年3月5日

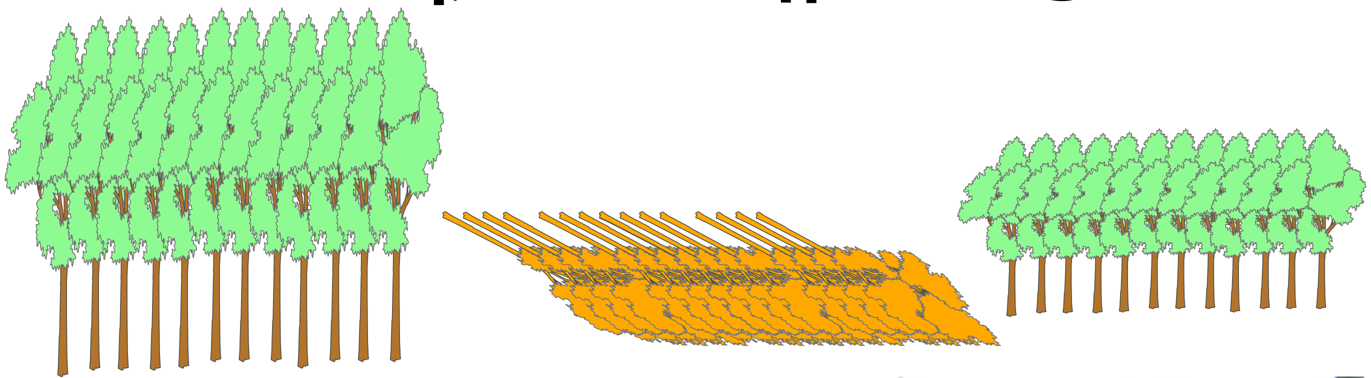
森林資源とデータ収集



古典的な森林資源の利用

- 木材生産

1．伐って植える



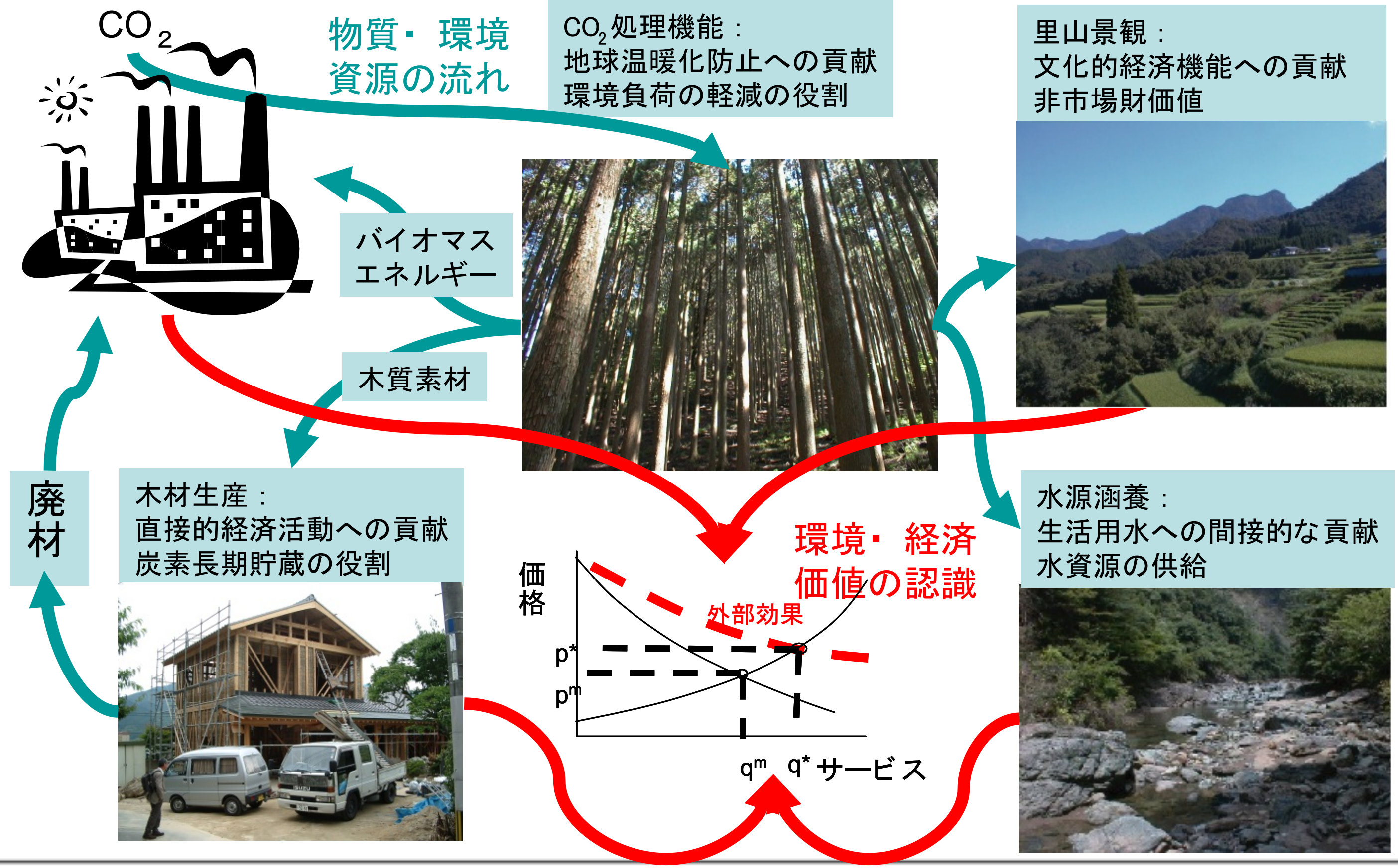
2．伐って使う

近代的な森林資源の利用・管理

森林の持つ多機能への認識

価値＝＞経済価値：生活スタイルの見直し

他の生産活動：
直接的経済活動
環境への負荷の発生



森林分野での最適化問題
-2つのターゲット-

林分レベル

複数の集まる森林レベル

対象とするスケールの違いにより対応が異なる

オペレーションズ・リサーチ2022年11月号

森林資源管理における最適化モデルの展開 (I)
—林分単位での最適化モデル—

吉本　敦，木島　真志

森林資源を管理するとき，管理の目的が必ず存在する．同時に自然環境あるいは経済社会環境に対するさまざまな制約も存在する．それらを把握し，適切な OR 手法の選択，モデルの構築，分析を通して管理者に対し意思決定を支援することが，OR 分野に携わる森林科学の研究者の主要な任務となる．これまで，森林資源を対象とした管理問題では，制約や制御の性質上，対象を大きく二つに分け対応してきた．一つが経営体の最小単位となる林分単位であり，もう一つが複数の林分からなる森林単位である．本稿では，これまで展開されてきた古典的な手法から近年の手法について，林分単位の最適化モデルに関連する研究に焦点を当てて著者の視点から詳説する．森林単位については次稿での「森林単位での最適化モデル」を参照されたい．

キーワード：最適間伐計画，成長予測，動的計画法

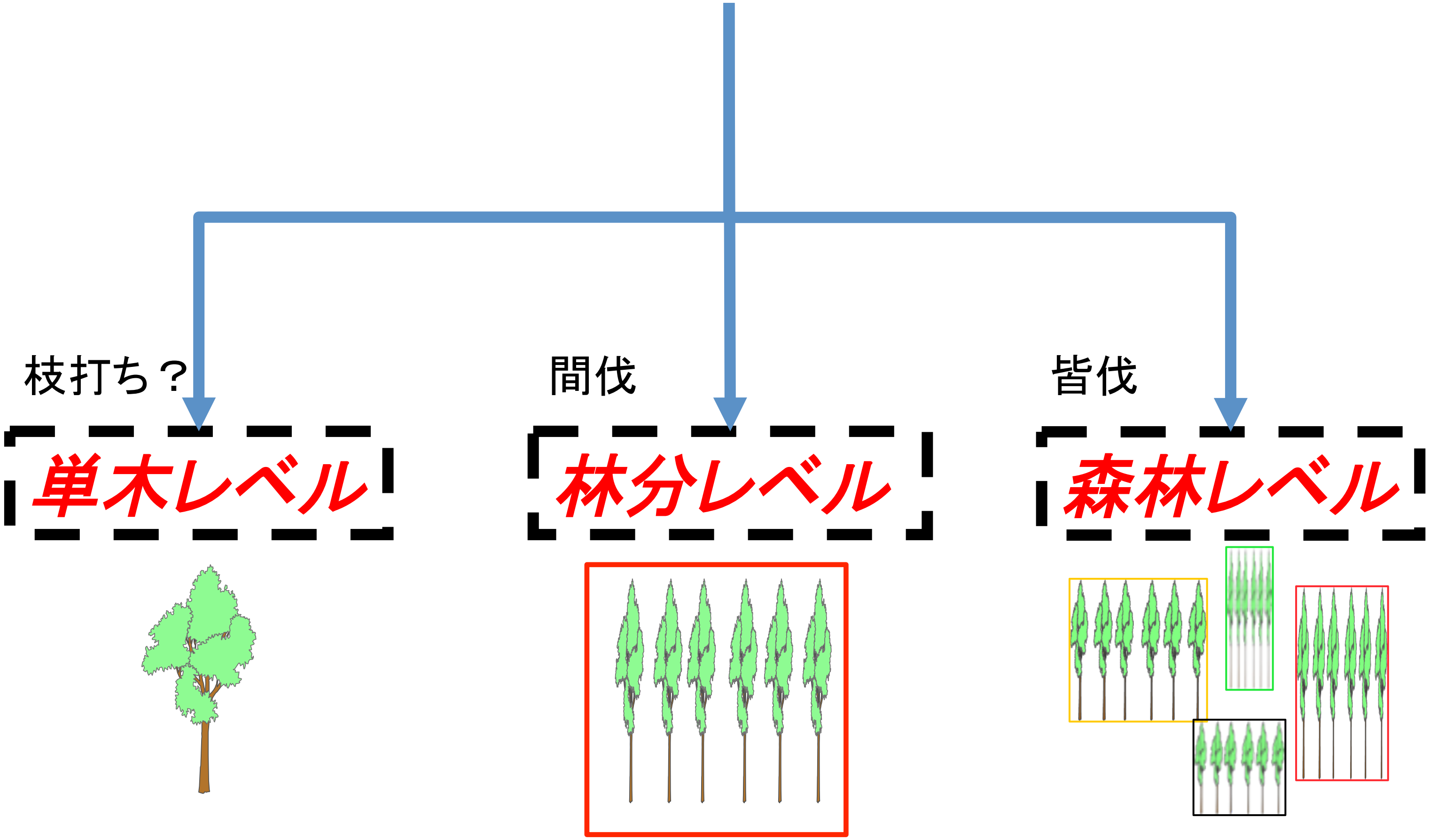
森林資源管理における最適化モデルの展開 (II)
—森林単位での最適化モデル—

吉本　敦，木島　真志

前稿では，経営体の最小単位となる"林分単位での最適化モデル"について詳説した．本稿では，複数の林分からなる森林単位の最適化モデルについて，これまで展開されてきた古典から近年まで，その関連研究について著者の視点から詳説する．

キーワード：伐採計画最適化，線形計画法，混合 0-1 整数計画法

森林分野での最適化問題
３つのターゲット？



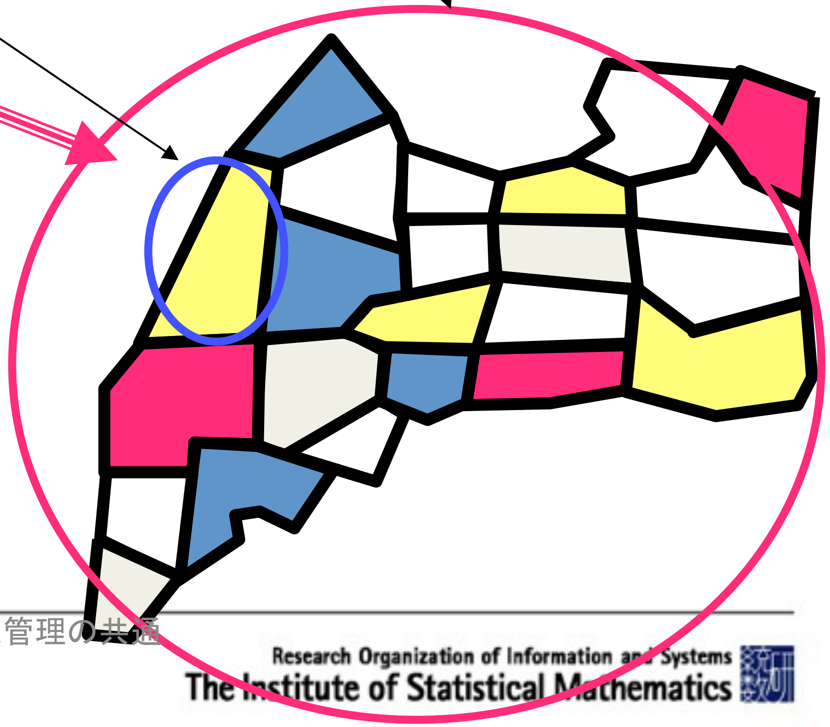
状態の分類

管理目的・制約が異なる

林分レベルでの管理

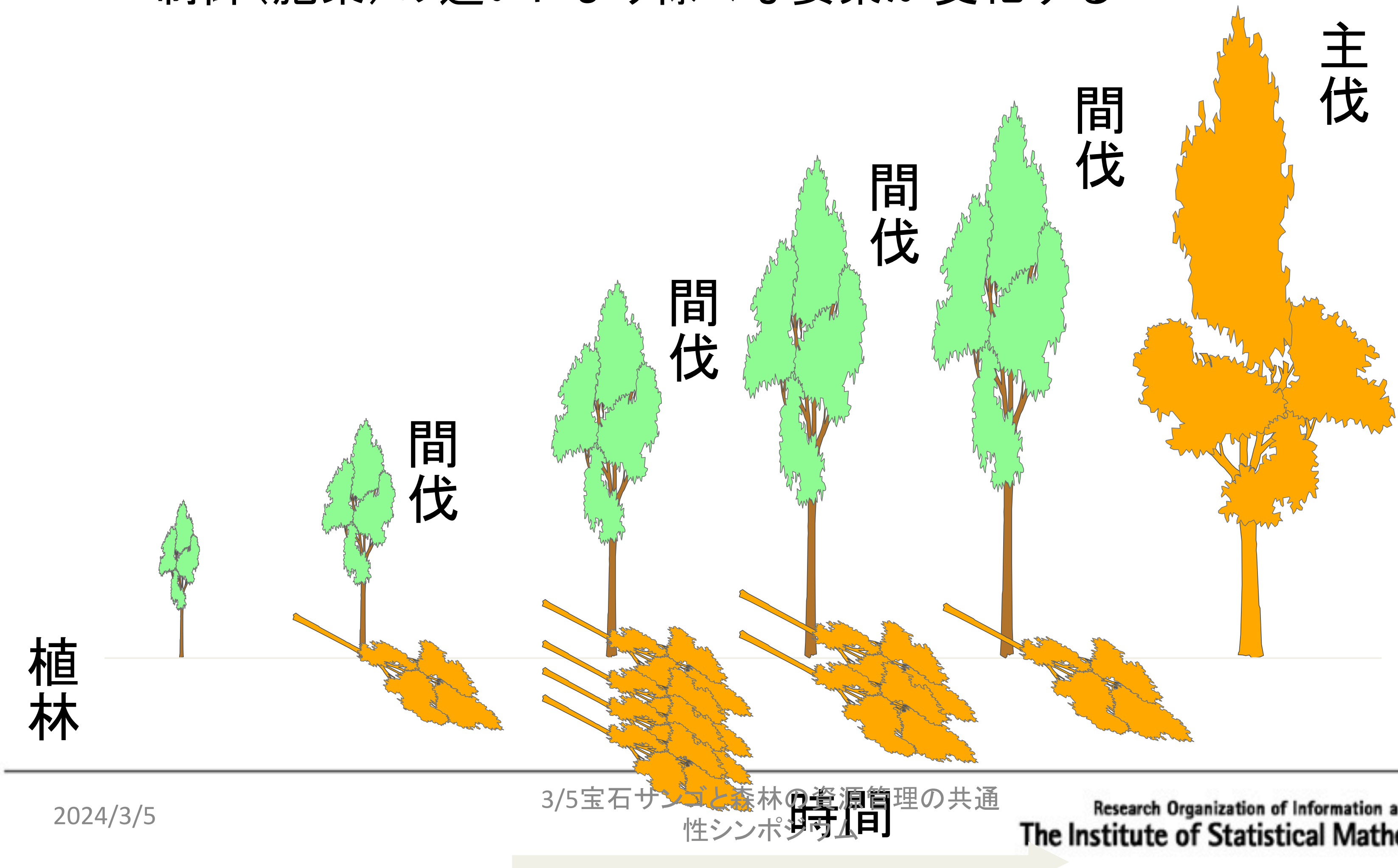
森林レベルでの管理

モデリングのための抽象化
・同一樹種林鈴、生産性など

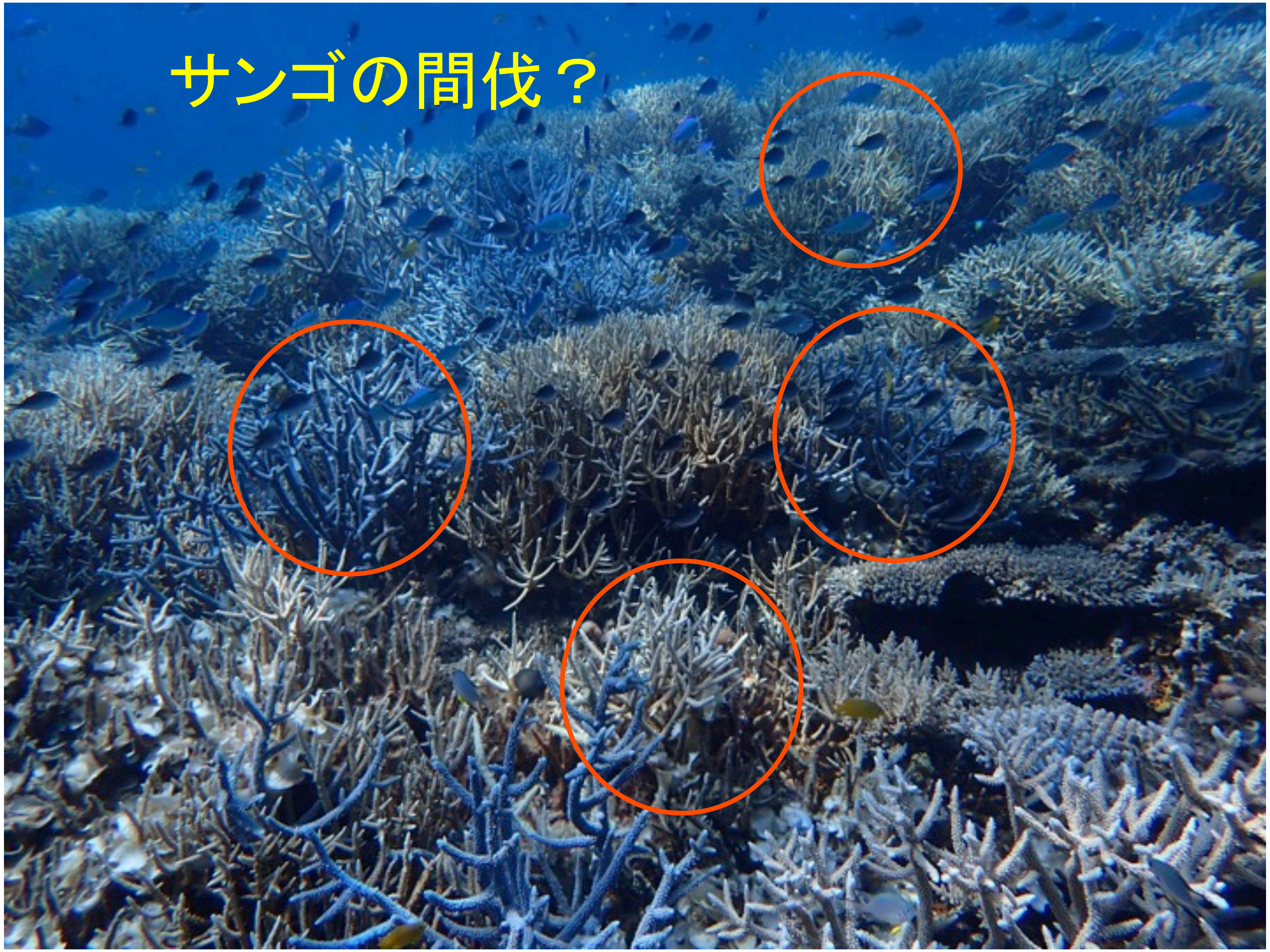


林分レベル
制御：間伐時期，強度，など

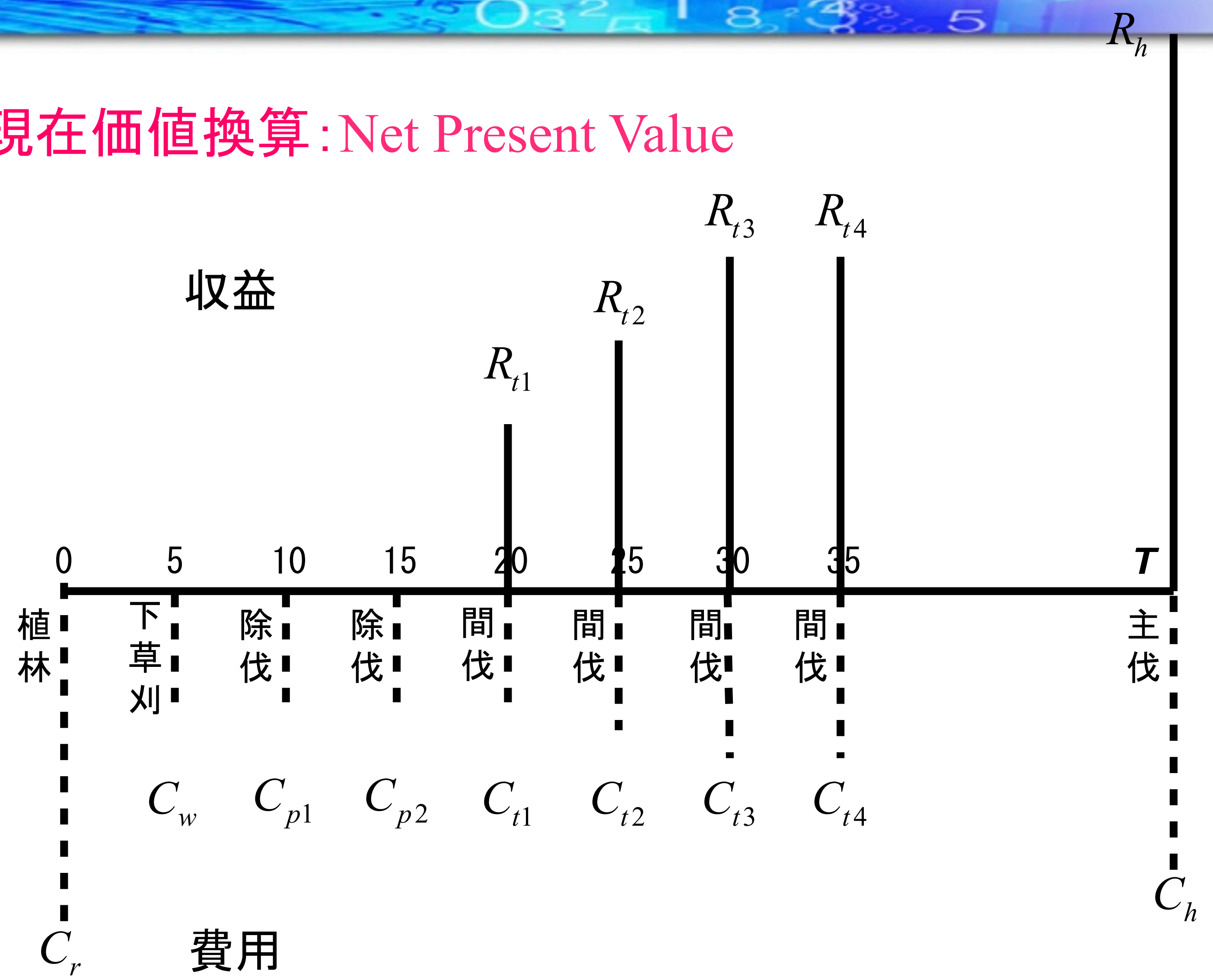
- ->制御（施業）の違いにより様々な要素が変化する



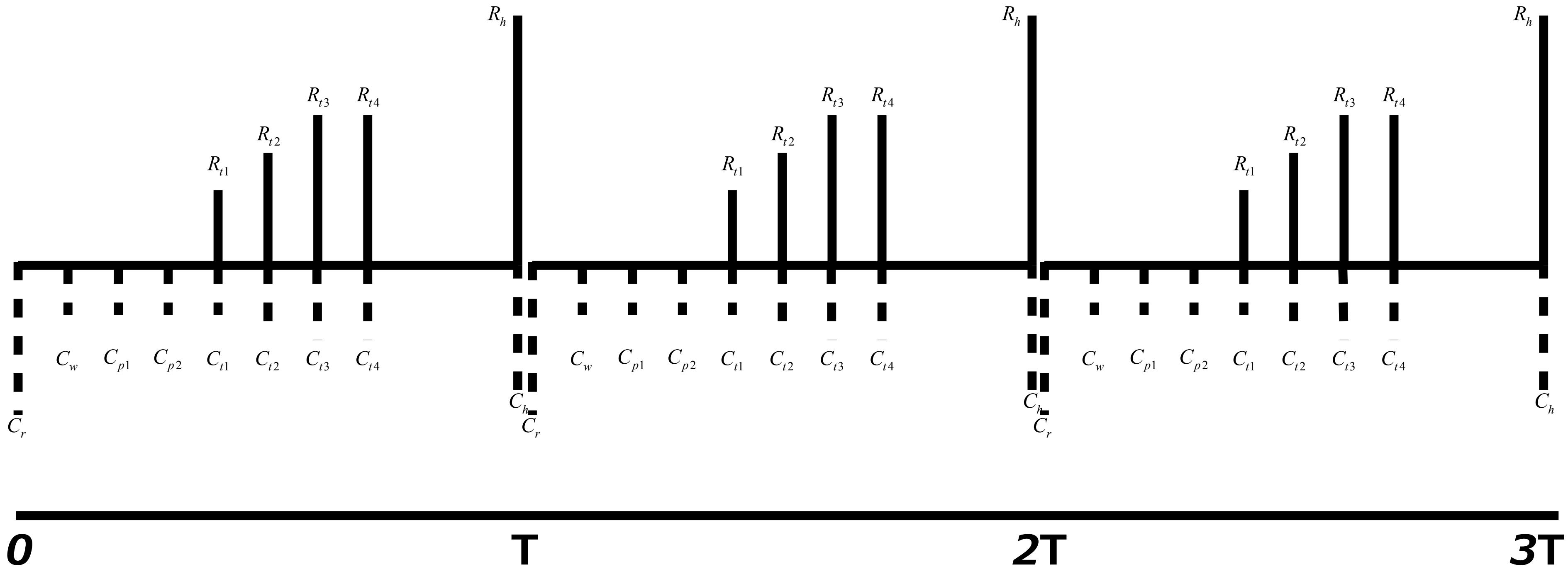
Spatially dependent individual growth model?



現在価値換算:Net Present Value



無限回の伐採: 土地期望価Soil Expectation Value



４） 土地期望価(Soil or Land Expectation Value: *SEV*)

$$NPV = -C_r - \frac{C_w}{(1+r)^5} - \frac{C_{p1}}{(1+r)^{10}} - \frac{C_{p2}}{(1+r)^{15}} + \frac{R_{t1}-C_{t1}}{(1+r)^{20}} + \frac{R_{t2}-C_{t2}}{(1+r)^{25}} + \frac{R_{t3}-C_{t3}}{(1+r)^{30}} + \frac{R_{t4}-C_{t4}}{(1+r)^{35}} + \frac{R_h-C_h}{(1+r)^T}$$

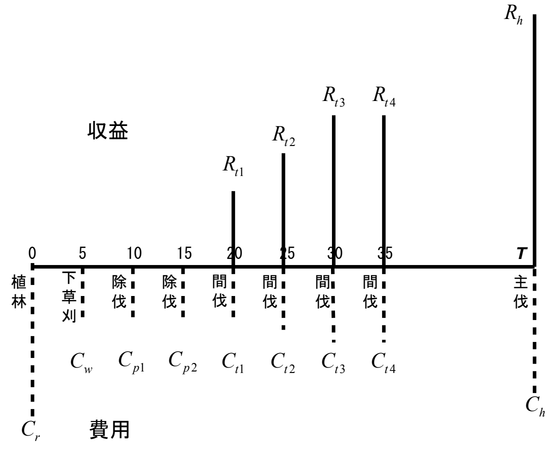


図１．一連の施業に関わる費用と収益

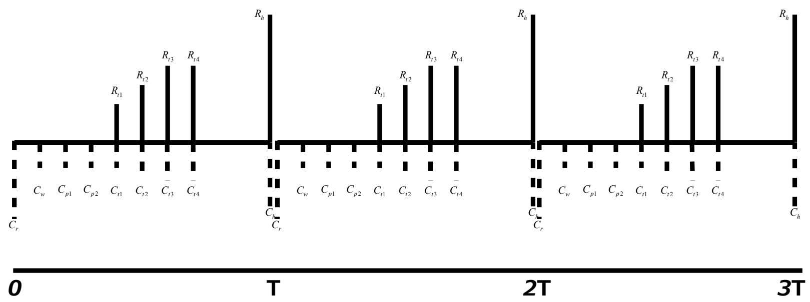
"Hossfeld formula" or known as "Faustmann formula"

$$SEV = NPV + \frac{NPV}{(1+r)^T} + \frac{NPV}{(1+r)^{2T}} + \frac{NPV}{(1+r)^{3T}} + \frac{NPV}{(1+r)^{4T}} \cdots\cdots\cdots$$

$$= \sum_{k=0}^{\infty} \frac{NPV}{(1+r)^{kT}}$$

$$= NPV \frac{(1+r)^T}{(1+r)^T - 1}$$

初項 NPV、公比 $(1+r)^{-T}$ の無限等比級数



林分レベル最適化
Bellmanの動的計画法の応用

元林業試験場の有水氏により世界に先駆けた動的計画法による間伐戦略最適化に関する林分経営の論文

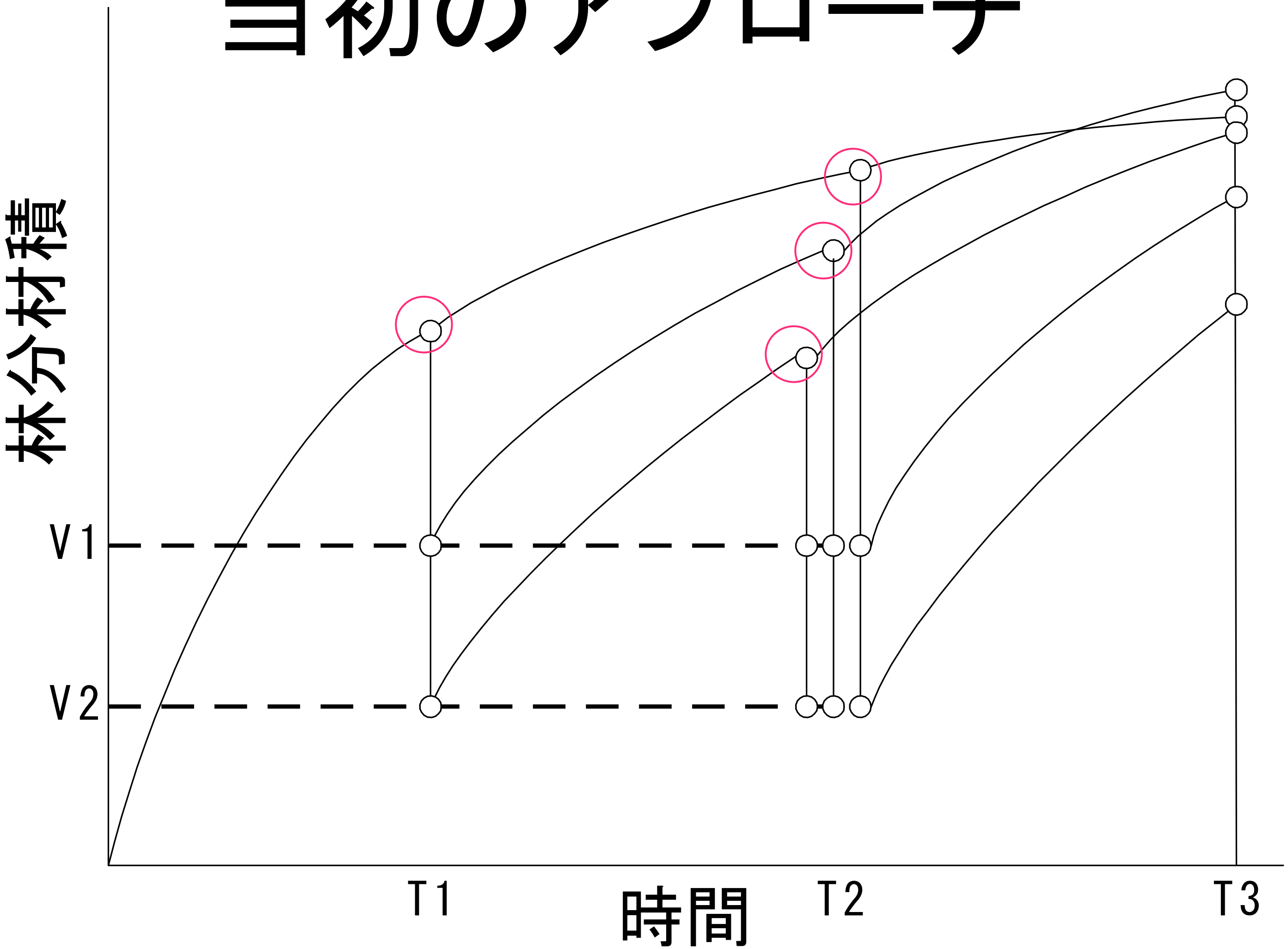
日本オペレーションズ・リサーチ学会誌(Arimizu 1958)

Arimizu T. Regulation of the cut by dynamic programming. J Oper Res Soc Jpn. 1958;1(4):175–82.

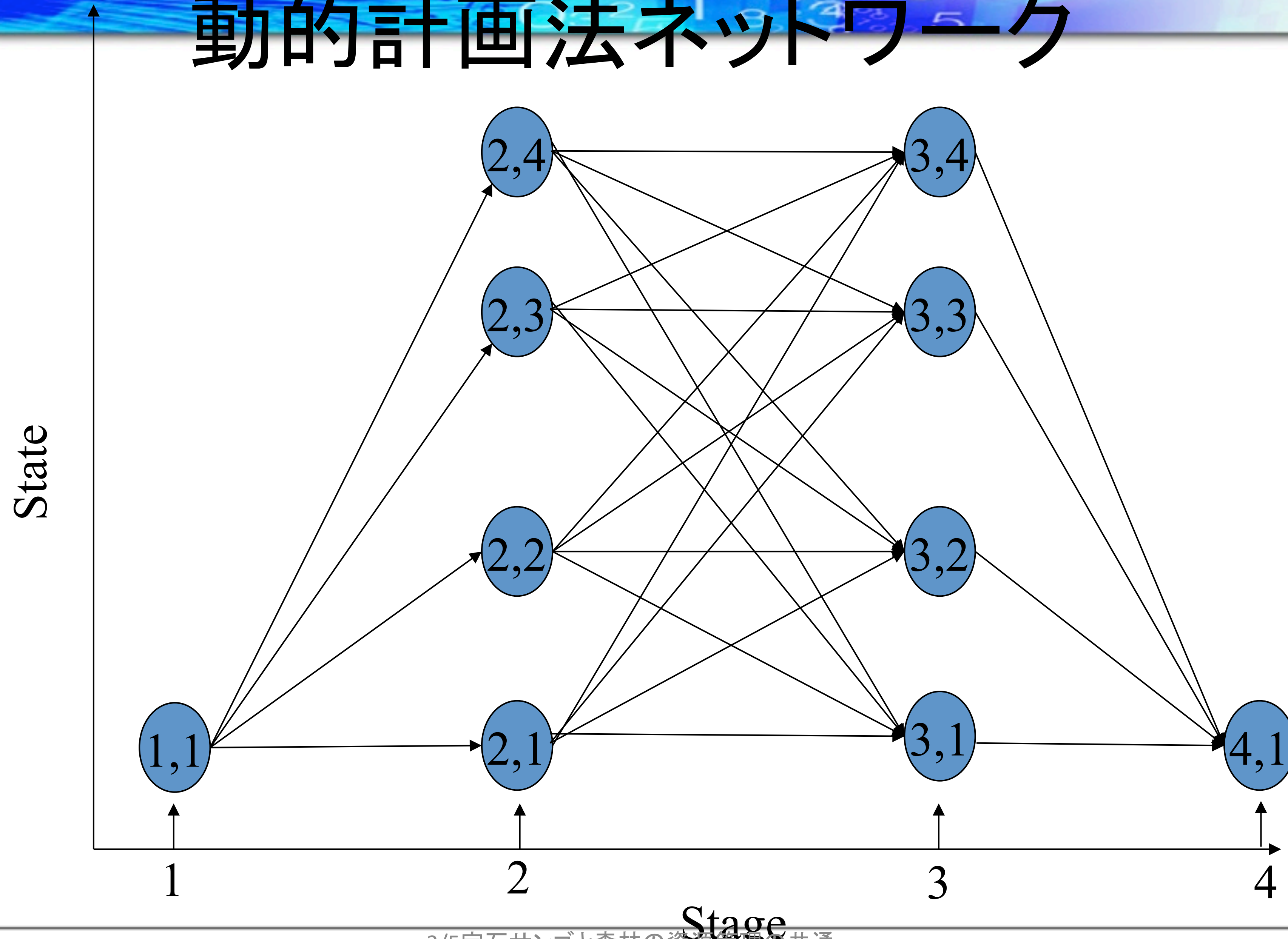
Table 1. Change in Optimization Approach over Years

Years	DP	NLP	Heuristic	Growth Model	Distance Type	Stand Type
1950s	Arimizu (1958)			Whole	Independent	Even-aged
1960s	Amidon & Akin (1968)			Whole	Independent	Even-aged
	Kilkki & Väisänen (1969)			Whole	Independent	Even-aged
1970s	Schreuder (1971)			Whole	Independent	Even-aged
		Adams & Ek (1974)		Age/stage structured models	Independent	Uneven-aged
	Brodie et al. (1978)			Whole	Independent	Even-aged
	Brodie & Kao (1979)			Whole	Independent	Even-aged
	Kao & Brodie (1979)			Whole	Independent	Even-aged

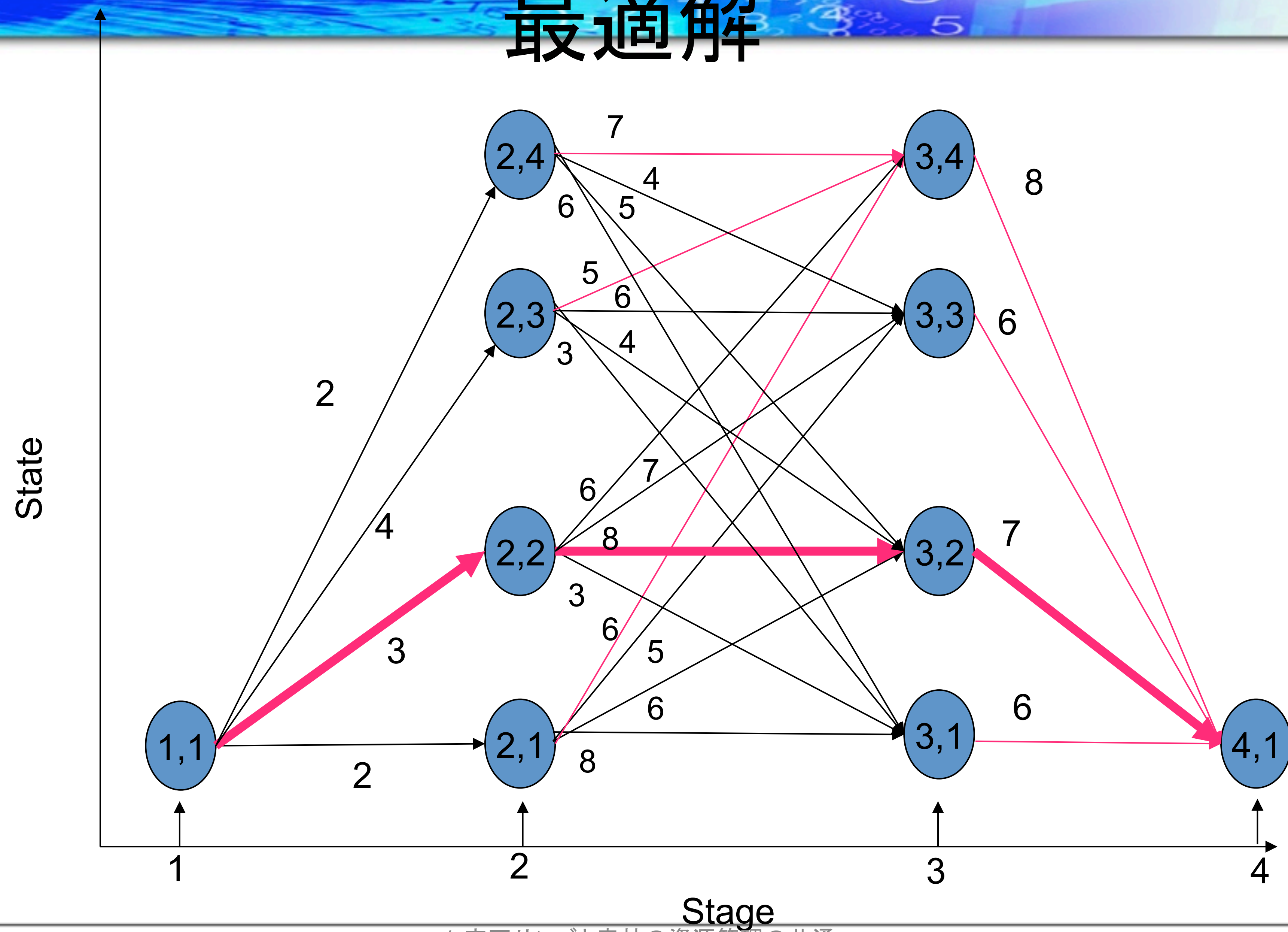
動的計画法:間伐戦略の探求
当初のアプローチ



動的計画法ネットワーク



最適解



State
Stage
1,1
2,1
2,2
2,3
2,4
3,1
3,2
3,3
3,4
4,1
1
2
3
4
2
4
3
2
7
4
6
5
5
6
3
4
6
7
8
3
6
5
6
8
8
6
7
6

Curr Forestry Rep (2016) 2:163–176
DOI 10.1007/s40725-016-0041-0

INTEGRATING FORESTRY IN LAND USE PLANNING (P BETTINGER, SECTION EDITOR)

Stand-Level Forest Management Planning Approaches

Atsushi Yoshimoto[1] · Patrick Asante[2] · Masashi Konoshima[3]

Dynamic Optimization

$$\max_{\{\mathbf{u}(t)\}} J = \int_{t_0}^{t_n} \dot{I}(\mathbf{x}(t), \mathbf{u}(t))\, dt$$

subject to

$$\dot{\mathbf{x}}(t) = f(\mathbf{x}(t), \mathbf{u}(t))$$

$$\mathbf{x}(t_0) = \mathbf{x}_0$$

成長（林分成長）

状態（林分の状態）

制御（間伐強度）

$\dot{I}$(x(t),u(t)):an instant performance index (NPV) from the current state of forest stand over the small interval of time

$f(\cdot)$: continuously differentiable function of $(\mathbf{x}(t), \mathbf{u}(t))$ to describe a dynamic change of the state $\mathbf{x}(t)$

$\mathbf{x}(t)$:a vector of time-varying state variables describing the state of a forest stand

$\mathbf{u}(t)$: a vector of control variables of thinning affecting the growth of a forest stand at time t

離散的な制御（間伐）への対応

$$\max_{\{\mathbf{u}(t_i)\}} J = \sum_{i=0}^{n-1} \int_{t_i}^{t_{i+1}} \dot{I}(\mathbf{x}(t), \mathbf{u}(t))dt$$

$$= \sum_{i=0}^{n-1} \{I(\mathbf{x}(t_{i+1}) \mid \mathbf{x}(t_i), \mathbf{u}(t_i)) - I(\mathbf{x}(t_i) \mid \mathbf{x}(t_i), \mathbf{u}(t_i))\}$$

$$\max_{\{\mathbf{u}(t_0), \ldots, \mathbf{u}(t_{n-1})\}} J = \sum_{i=0}^{n-1} \{I(\mathbf{x}(t_{i+1}) \mid \mathbf{x}(t_i), \mathbf{u}(t_i)) - I(\mathbf{x}(t_i) \mid \mathbf{x}(t_i), \mathbf{u}(t_i))\}$$

$$= \sum_{i=0}^{n-1} \{I(\mathbf{x}(t_{i+1}) \mid \mathbf{x}(t_i), \mathbf{u}(t_i)) + I(\mathbf{u}(t_i)) - I(\mathbf{x}(t_i) \mid \mathbf{x}(t_{i-1}), \mathbf{u}(t_{i-1}))\}$$

subject to

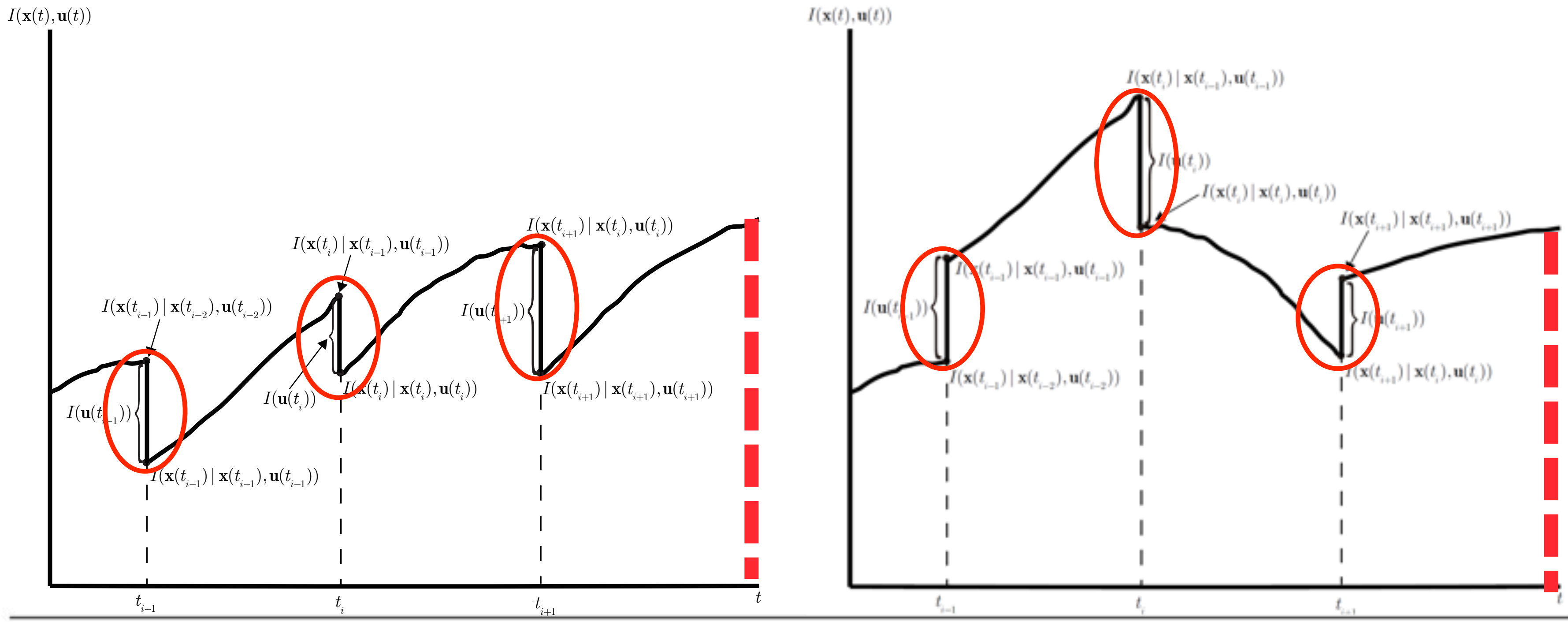
$$\mathbf{x}(t_{i+1}) = g(\mathbf{x}(t_i), \mathbf{u}(t_i))$$

$$I(\mathbf{x}(t_i) \mid \mathbf{x}(t_i), \mathbf{u}(t_i)) = I(\mathbf{x}(t_i) \mid \mathbf{x}(t_{i-1}), \mathbf{u}(t_{i-1})) - I(\mathbf{u}(t_i))$$

Graphical Expression

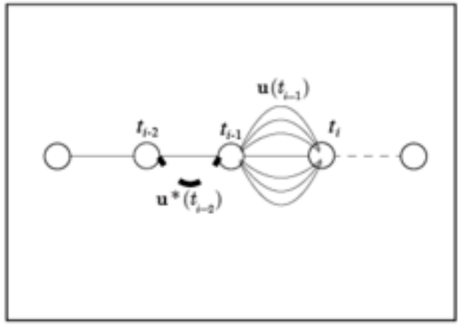
$$\max_{\{\mathbf{u}(t_0),\ldots,\mathbf{u}(t_{n-1})\}} J = \sum_{i=0}^{n-1} \{I(\mathbf{x}(t_{i+1}) \mid \mathbf{x}(t_i), \mathbf{u}(t_i)) - I(\mathbf{x}(t_i) \mid \mathbf{x}(t_i), \mathbf{u}(t_i))\}$$

$$= \sum_{i=0}^{n-1} \{I(\mathbf{x}(t_{i+1}) \mid \mathbf{x}(t_i), \mathbf{u}(t_i)) + I(\mathbf{u}(t_i)) - I(\mathbf{x}(t_i) \mid \mathbf{x}(t_{i-1}), \mathbf{u}(t_{i-1}))\}$$



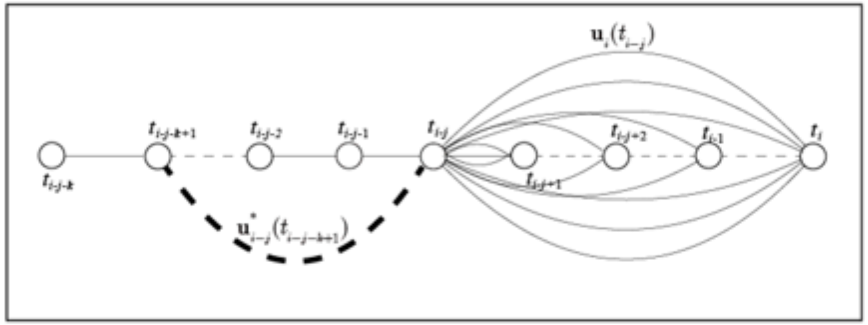
One-State and One-Stage DP Network
制御効果の評価

PATH



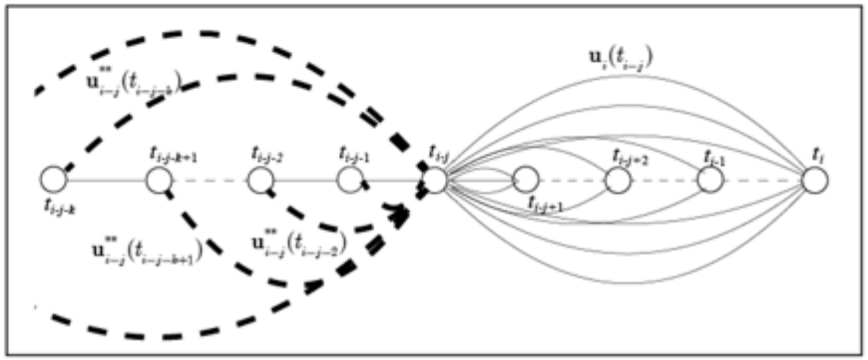
One-Stage Look-Ahead

MSPATH



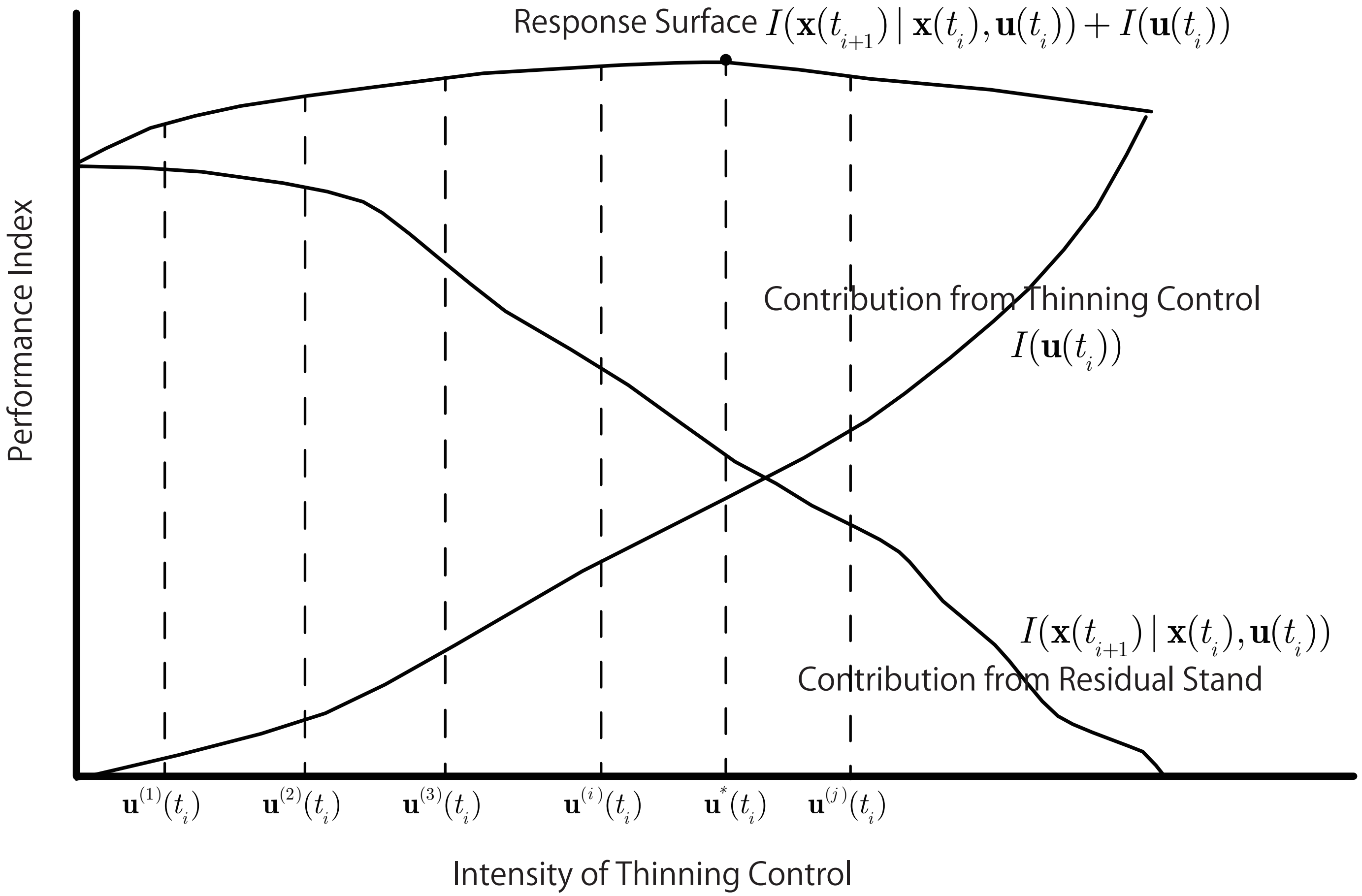
Multi-Stage Look-Ahead

TMSPATH



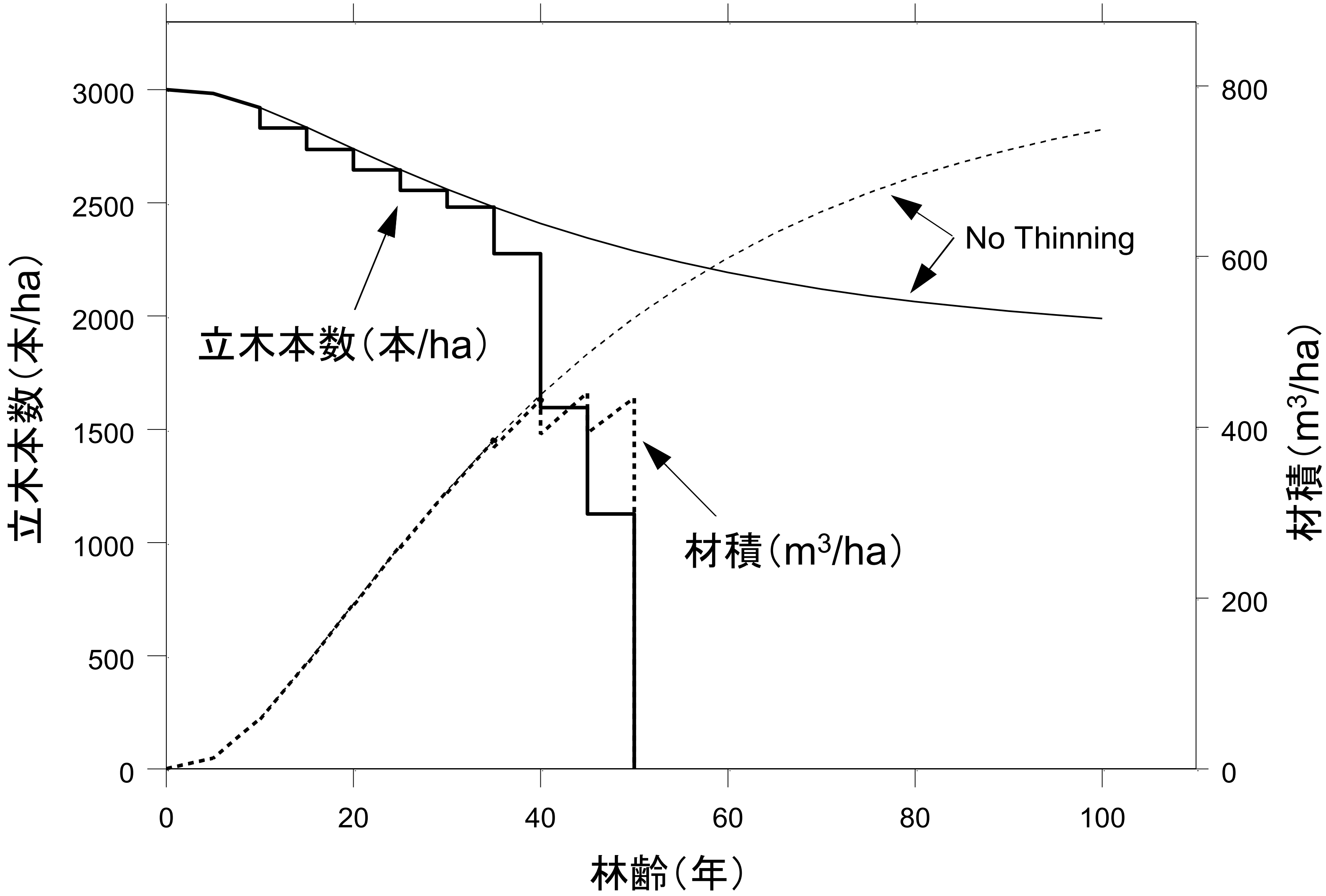
Two-directional Multi-Stage Look-Ahead

制御と反応面



Response Surface $I(\mathbf{x}(t_{i+1}) \mid \mathbf{x}(t_i), \mathbf{u}(t_i)) + I(\mathbf{u}(t_i))$
Performance Index
Contribution from Thinning Control
$I(\mathbf{u}(t_i))$
$I(\mathbf{x}(t_{i+1}) \mid \mathbf{x}(t_i), \mathbf{u}(t_i))$
Contribution from Residual Stand
$\mathbf{u}^{(1)}(t_i)$
$\mathbf{u}^{(2)}(t_i)$
$\mathbf{u}^{(3)}(t_i)$
$\mathbf{u}^{(i)}(t_i)$
$\mathbf{u}^{*}(t_i)$
$\mathbf{u}^{(j)}(t_i)$
Intensity of Thinning Control

最適間伐戦略



3000
2500
2000
1500
1000
500
0
800
600
400
200
0
0
20
40
60
80
100
立木本数（本/ha）
材積（m3/ha）
立木本数（本/ha）
材積（m3/ha）
No Thinning
林齢（年）

林分成長モデル

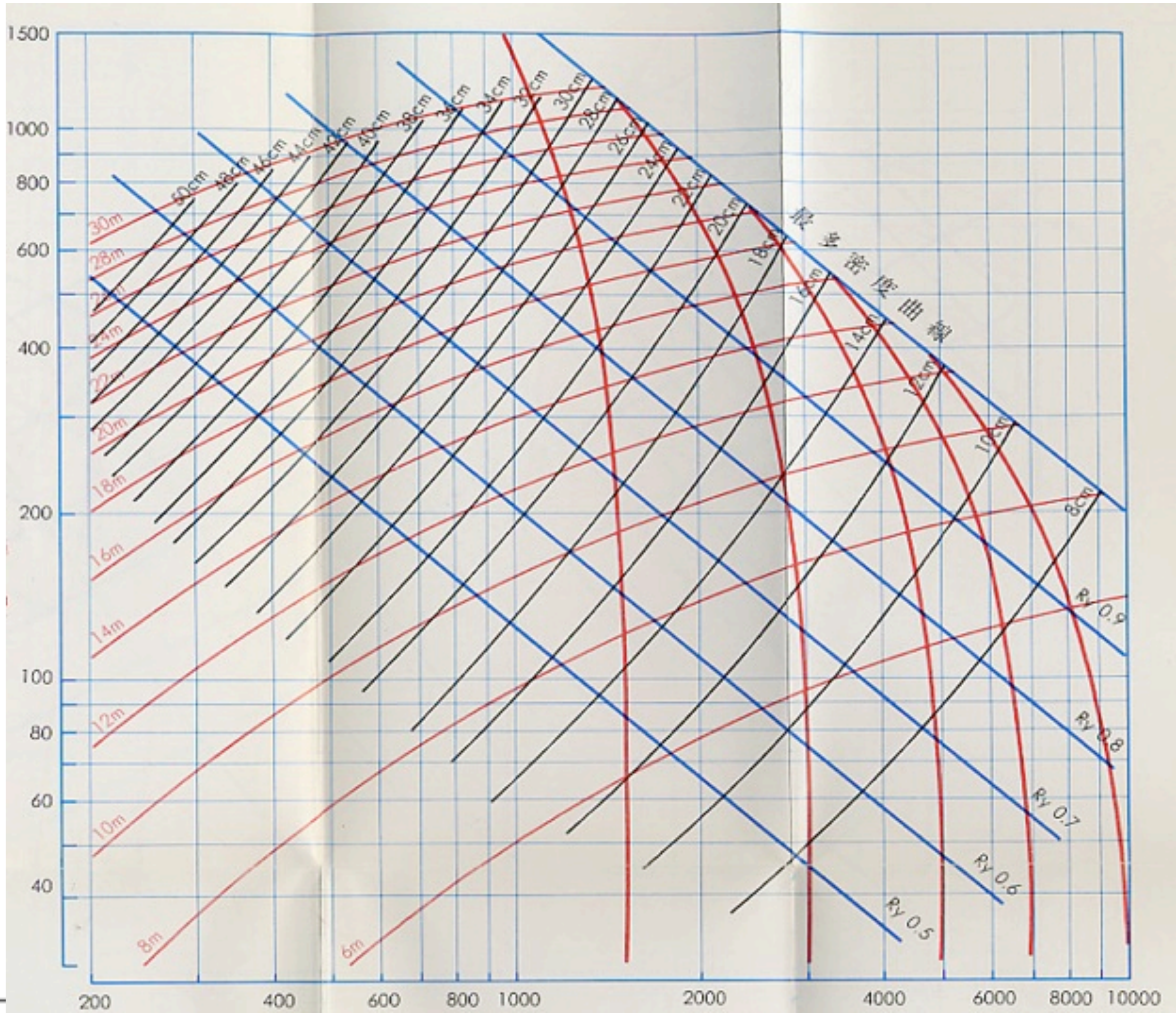
↓

動的計画法

↓

持続的林分管理の最適化

林分の成長モデル（密度管理図）：状態依存 胸高直径，樹高，材積，立木密度**(Ando 1968)**



林分材積 (m^3/ha)

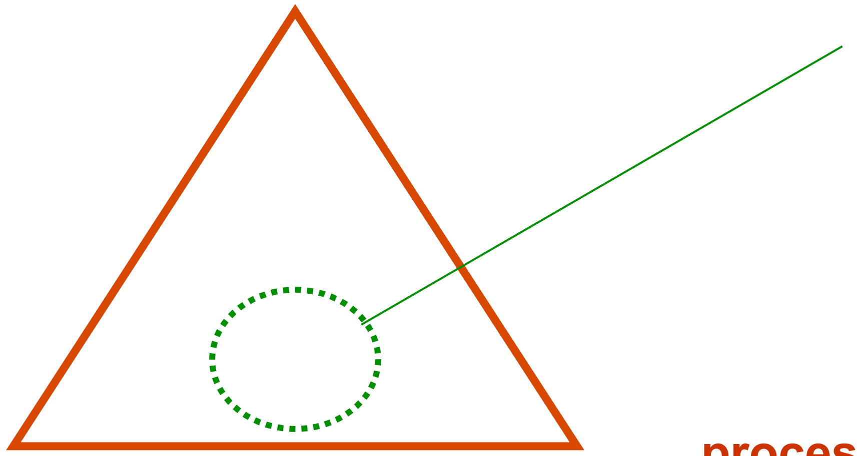
立木密度(本/ha)

密度管理図の関数群

平均単木材積	$v = \dfrac{1}{0.068509N \cdot H^{-1.347464} + 2658.2 \cdot H^{-2.814651}}$
自然枯死線 植栽本数 N_0	$\dfrac{1}{N} = \dfrac{1}{N_0} + \dfrac{v}{3.47089 \times 10^6 N_0^{-0.9184}}$
ha当たり材積	$V = v \cdot N$
林分形状高	$HF = 0.791213 + 0.244012H\sqrt{N}/100 + 0.353895H$
ha当たり断面積	$G = \dfrac{V}{HF}$
断面積平均直径	$\overline{D}g = 200\sqrt{G/(\pi \cdot N)}$
平均胸高直径	$\overline{DBH} = -0.048940 - 0.034814H\sqrt{N}/100 + 0.98937Dg$
最多密度におけるha当たり本数	$N_{Rf} = 10^{5.3083}H^{-1.4672}$
最多密度におけるha当たり材積	$V_{Rf} = \dfrac{N_{Rf}}{0.068509N_{Rf} \cdot H^{-1.347464} + 2658.2H^{-2.814651}}$
収量比数	$Ry = \dfrac{V}{V_{Rf}}$

Classification of forest models (Kurth, 1994):

Empirical correlation models, Process (mechanism) models, Visual (structural) models



Statistical (correlation) models: predicted variable is modelled on predicting plant variable
(height on diameter, diameter on age)

Process models: predicted variable is modelled as result of processes: photosynthesis,
light interception (upscale to higher level), respiration etc.

Structural models: predicted variable is the structure and visual look
(behind the structure there is a growth algorithm)

Functional Structural Plant Models :

•Linking of botanical structures <u>and</u> functions (e.g., light interception, water flow) in a coherent, single model

• processes linked to morphological objects

History :

• Dynamical description of structures

• Algorithmization of plant growth

L-systems (Lindenmayer systems):

• rule systems for the replacement of character strings

• in each derivation step parallel replacement of all characters for which there is one applicable rule

• by A. Lindenmayer (botanist) introduced in 1968 to model growth of filamentous algae

Aristid Lindenmayer (1925-1989)

The idea is that the plant is the result of sequence of stages if we interfere in one the rest is going to be influenced

Example:

Variables: A, B

Start : A

Set of rules:

A → B
B → AB

Time interval (1 day/ 1 year)

derivation chain for algae:

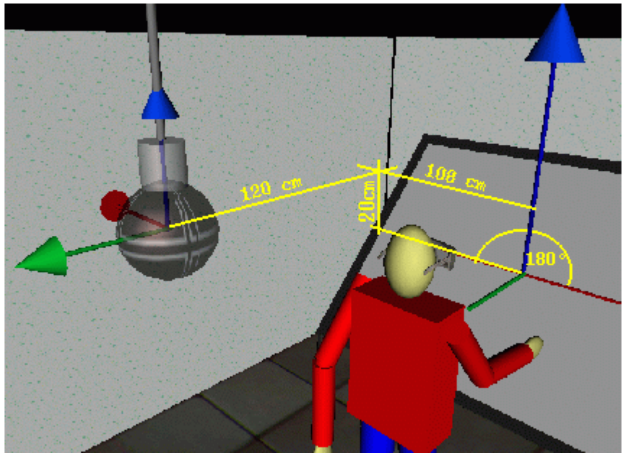
A → B → AB → BAB → ABBAB → BABABBAB → ABBABBABABBAB
→ BABABBABABBABBABABBAB → ...

Inputs are 3D measure data:

Magnetic motion tracker

+ cheap (relatively)
+ offers directly relative coordinates
+ magnetic field passes “through” material

- no metalic object
- weather
- small amount of works (communication+math)



- Dificult is to expose the object (gain access to points)

- Digitalization - relatively easy

- Problems to overcome:

1 Heat problem

2 Rain problem

3 Size problem

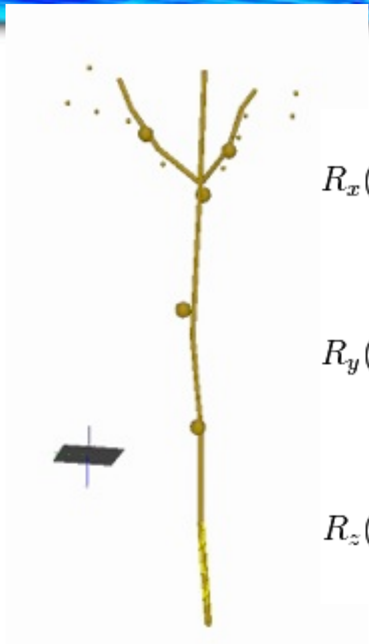
People are
giving up ...

Heat problem

Project planning...

Rain problem, size problem

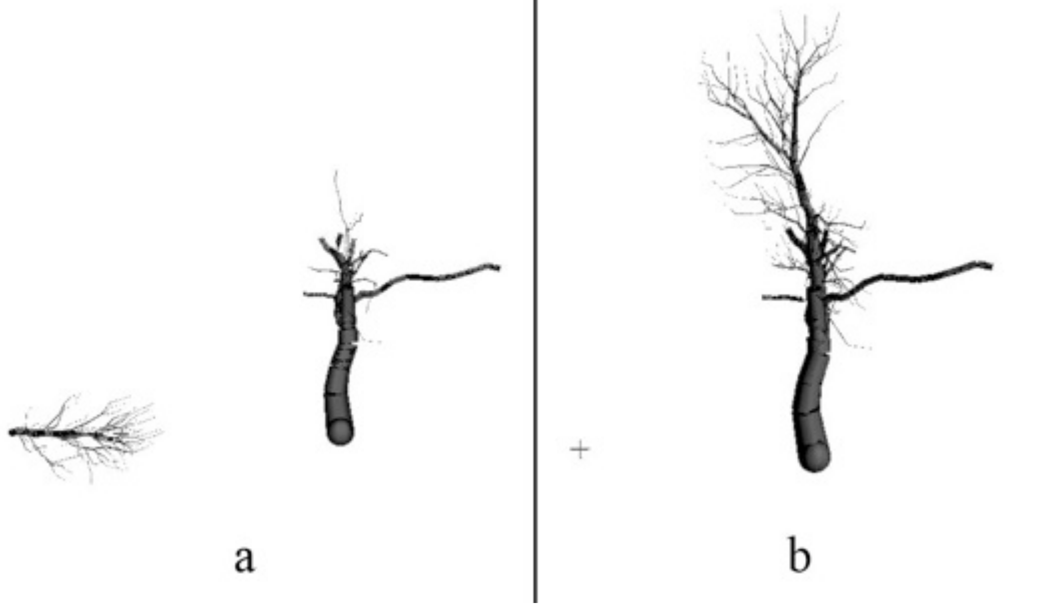
Rain problem, size problem



$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

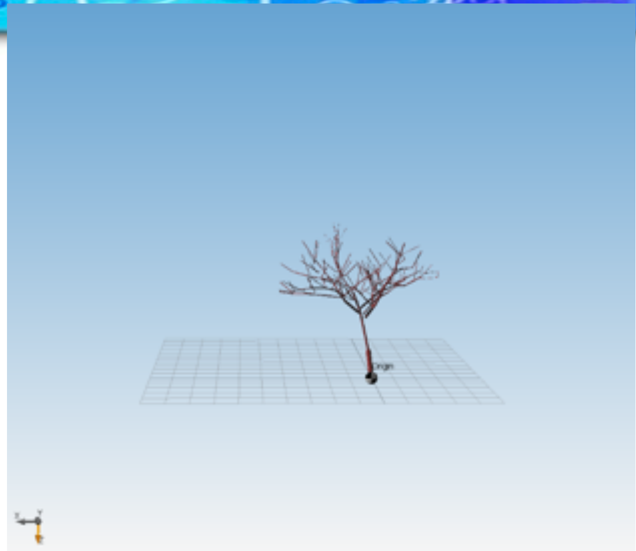
$$R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

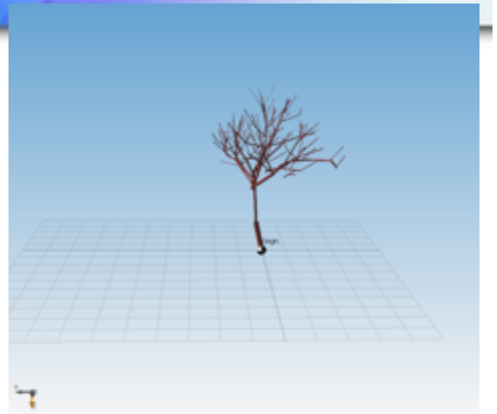
$$R_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$



$$x = \begin{pmatrix} 12.55 & 12.46 & 3.37 \\ 3.91 & 0.55 & 0.55 \\ 11.19 & -21.35 & -16.26 \\ -8.44 & -15.71 & -17.42 \\ -23.65 & -7.13 & 3.36 \end{pmatrix} \qquad y = \begin{pmatrix} -7.89 & -100.25 & -49.17 \\ -9.04 & -86.37 & -54.68 \\ -12.67 & -62.5 & -38.9 \\ -22.61 & -62.61 & -56.9 \\ -12.7 & -73.57 & -79.53 \end{pmatrix}$$

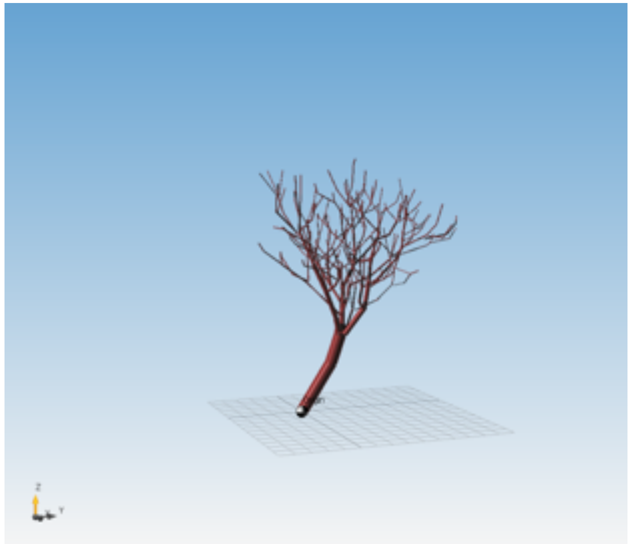
$$k = \begin{pmatrix} 0.3099 & -0.2497 & 0.9252 & 35.5535 \\ -0.3812 & -0.9158 & -0.1001 & -87.3493 \\ 0.8505 & -0.3040 & -0.4007 & -40.0436 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

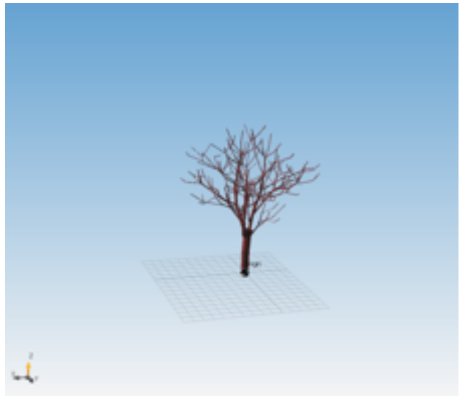




Mehirugi

total length	**12.71004**
total volume	**250.1523**



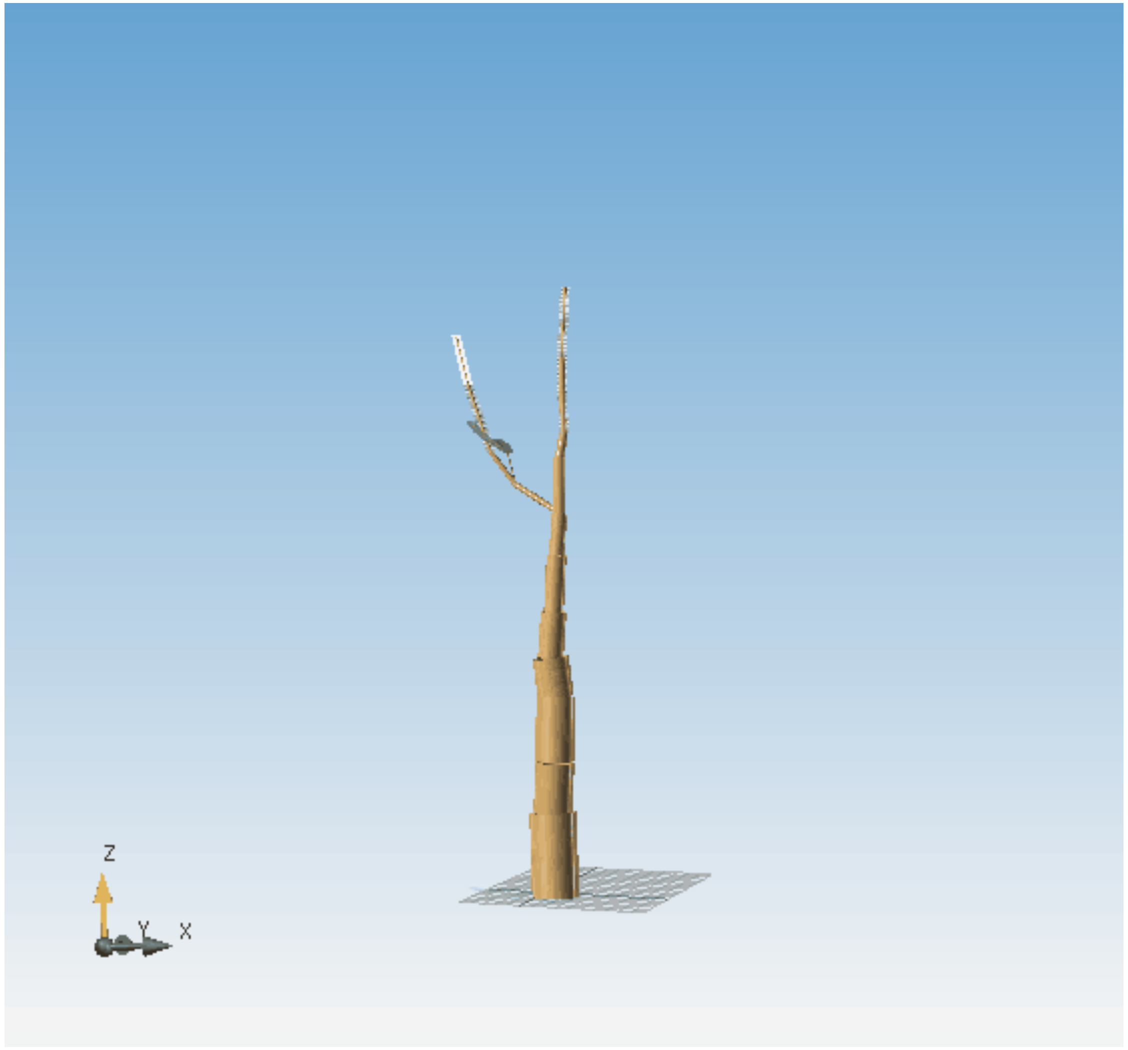


Ohirugi

total length	**36.94371**
total volume	**6419.577**

Length and volume estimated from mensuration in m2 and cm3

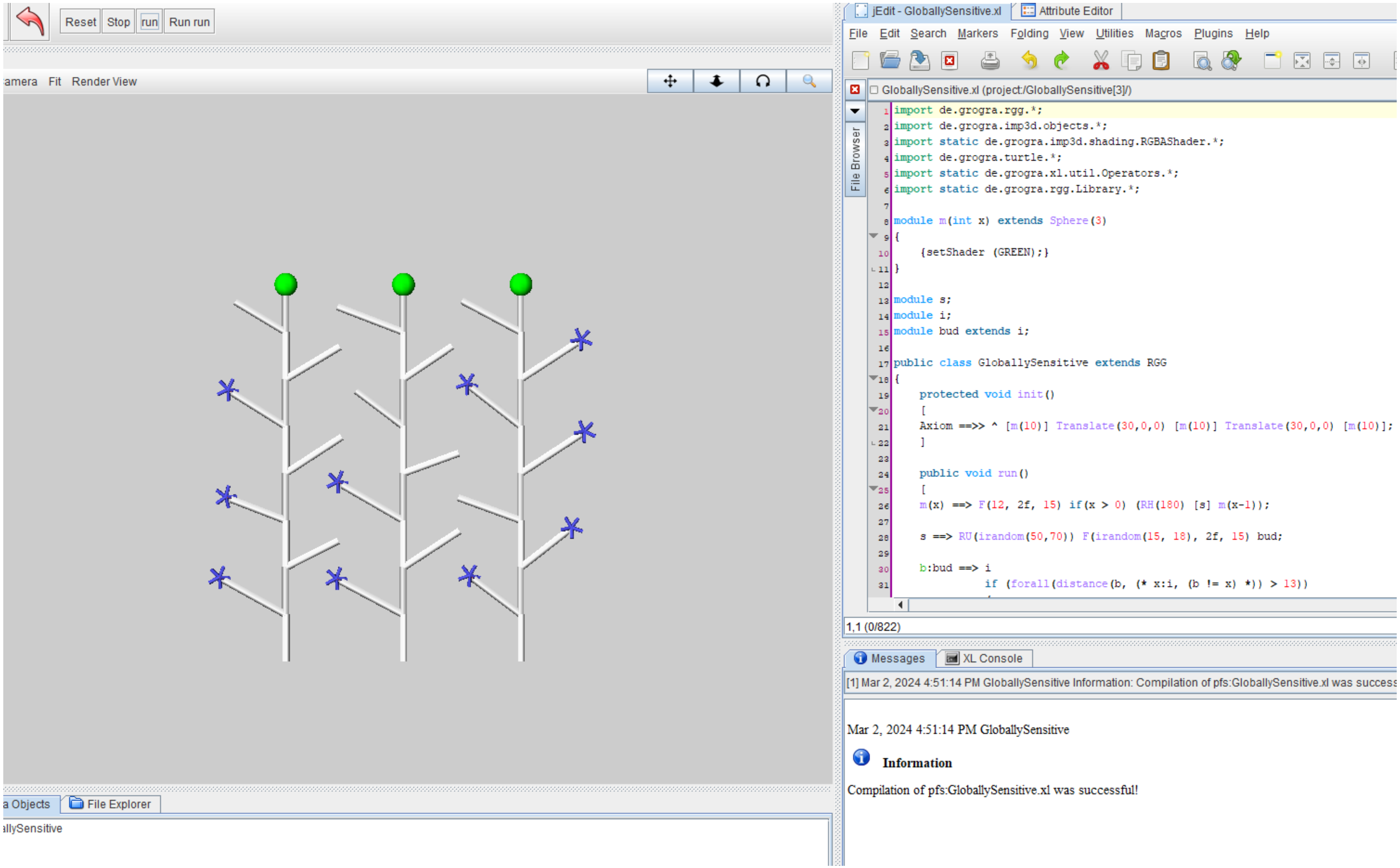
Animation of how the above ground structure is being measured

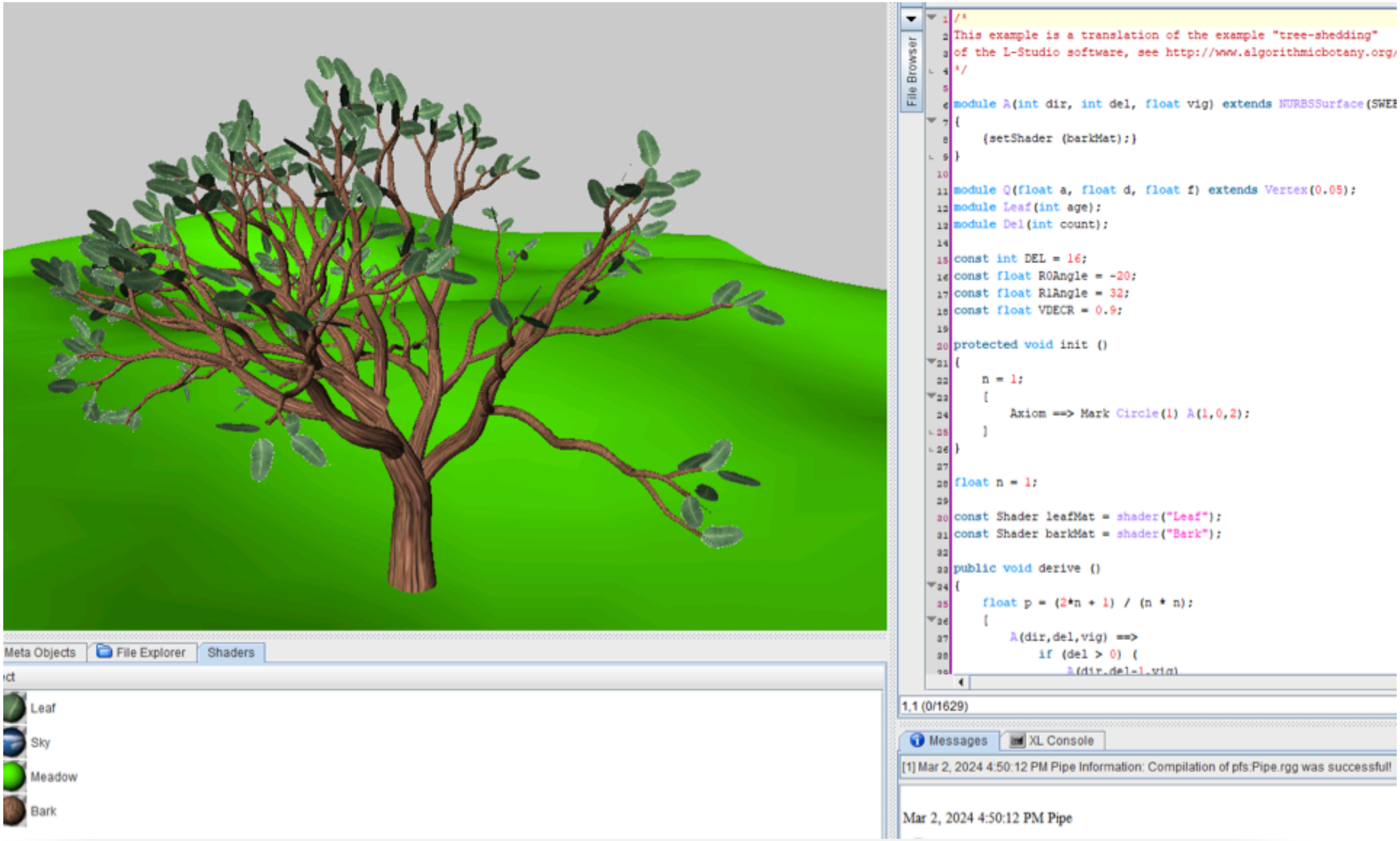


Animation of how the above ground structure is being measured

Fixation in terrain allows the mensuration per partes,
e.g. individual branches and roots can be removed after they are digitized but the system can be visualize together

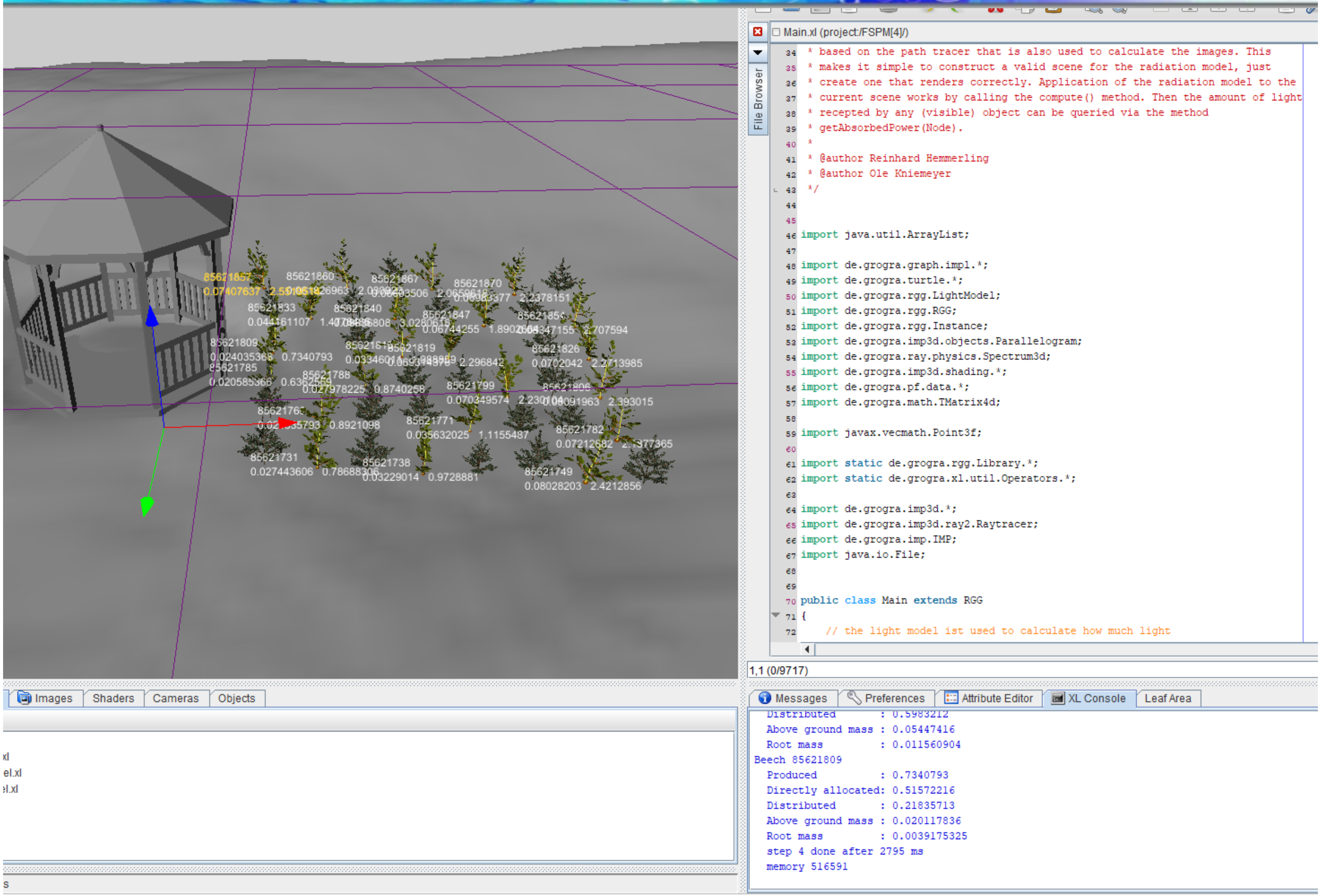
Data are modelled using growth models of functional structural models and growth algorithms. Here example of obstacle detection in growth directory (no flower)





Code for surfaces and textures

Simulation of plantation forest. Right bottom outputs about biomass, carbon uptake etc.



Toward 3D Application

Ashi Yoshimoto & Peter Surovy
Institute of Statistical Mathematics, Tokyo
JAPAN

Creation of MESH

Point field data has to be analyzed and meta processed
Interpolation is one possibility though
Interpolation creates information which is not measured
And eliminates information which is present

There exist 3 groups for MESH reconstruction from point field

- **Sculpting-based approaches**

Boissonnat J-D. Geometric structures of three-dimensional shape reconstruction. ACM Trans. Graphics 1984;3(4):266–86.

- **Contour-tracing approaches**

Hoppe H, DeRose T, Duchamp T, McDonald J, Stuelzle W. Surface reconstruction from unorganized points. Comput Graphics (Proc SIGGRAPH'92) 1992;71–8.

- **Region-growing approaches**

Huang J, Menq CH. Combinatorial manifold mesh reconstruction and optimization from unorganized points with arbitrary topology. Comput-Aided Des 2002;(34):149–65

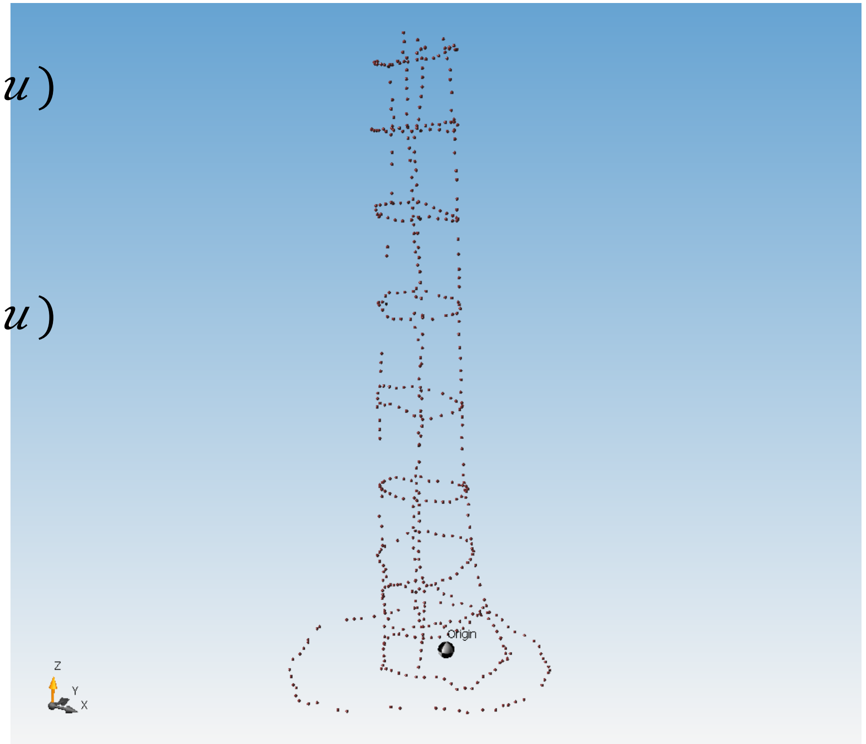
these are basically understood as optimization problems

Horizontal distance

$$P_n = \min(||P_{n-1,u}, P_{n,b}||, ||P_{n-1,b}, P_{n,u})$$

3D distance

$$P_n = \min(||P_{n-1,u}, P_{n,b}||, ||P_{n-1,b}, P_{n,u})$$

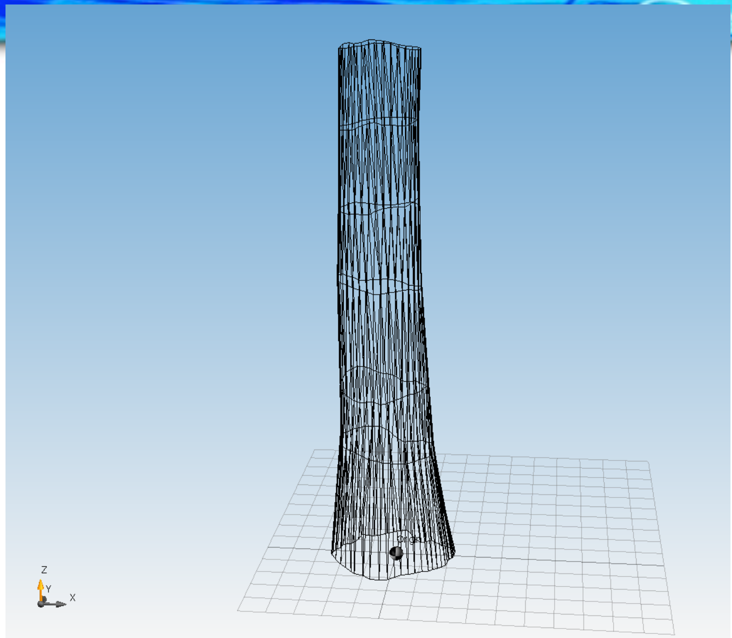


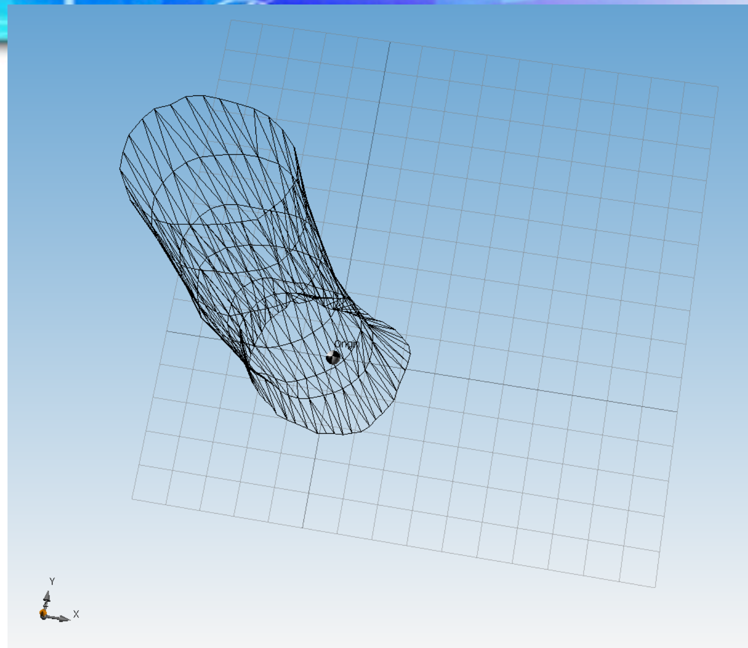
Application of region growth method

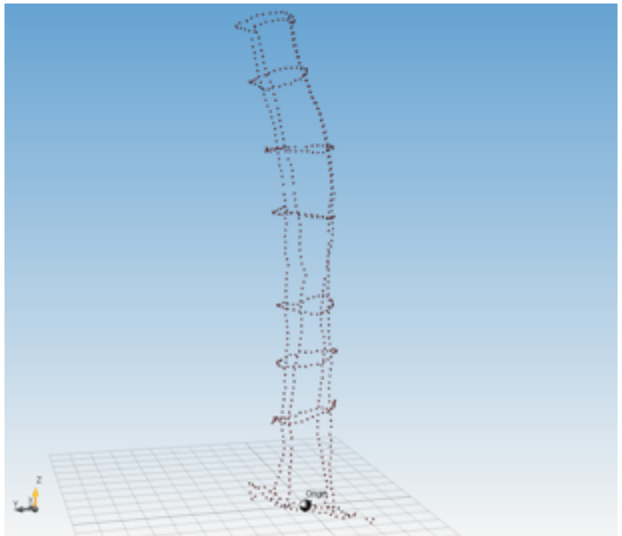
+ works well for horizontally aligned circles
+ Easiest to apply

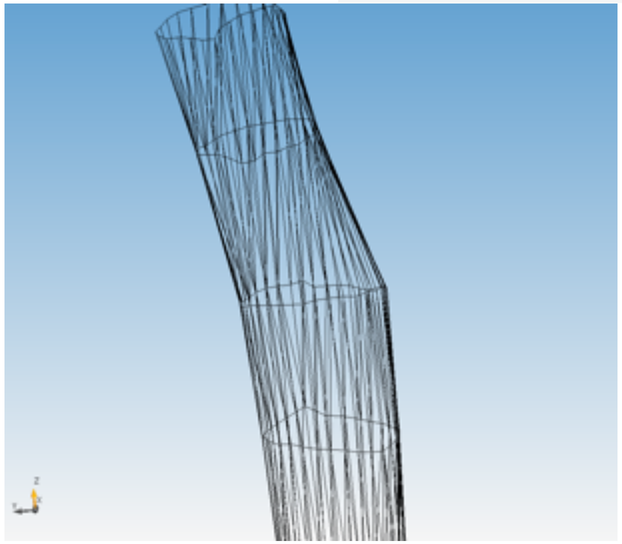
- Fails to work on unexpected non horizontal points

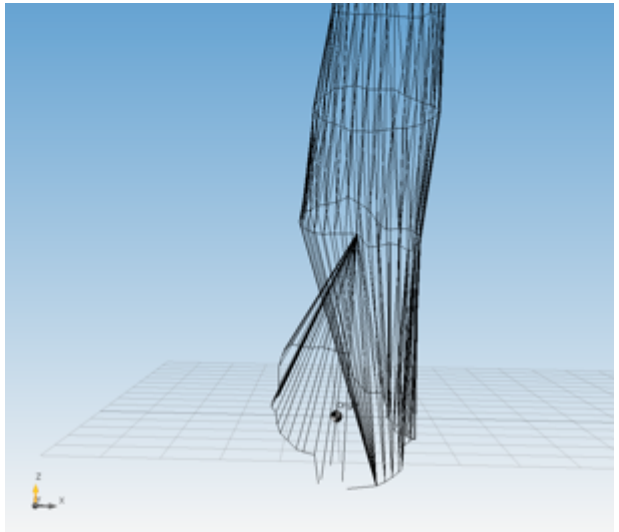
Minimum distance











Minimum angle of

$$P_n = \min(\tan(r,u), \tan(q,v))$$

Where:
r is a plane given by equation :

$$(x,y,z) = (x_0 + \lambda u_1 + \mu v_1, y_0 + \lambda u_2 + \mu v_2, z_0 + \lambda u_3 + \mu v_3)$$

Where

$$P_{n-1,u} = (x_0, y_0, z_0)$$

$$u_1 = (P_{n-1,u}, P_{n,u})$$

$$v_1 = (P_{n-1,u}, P_{n-1,b})$$

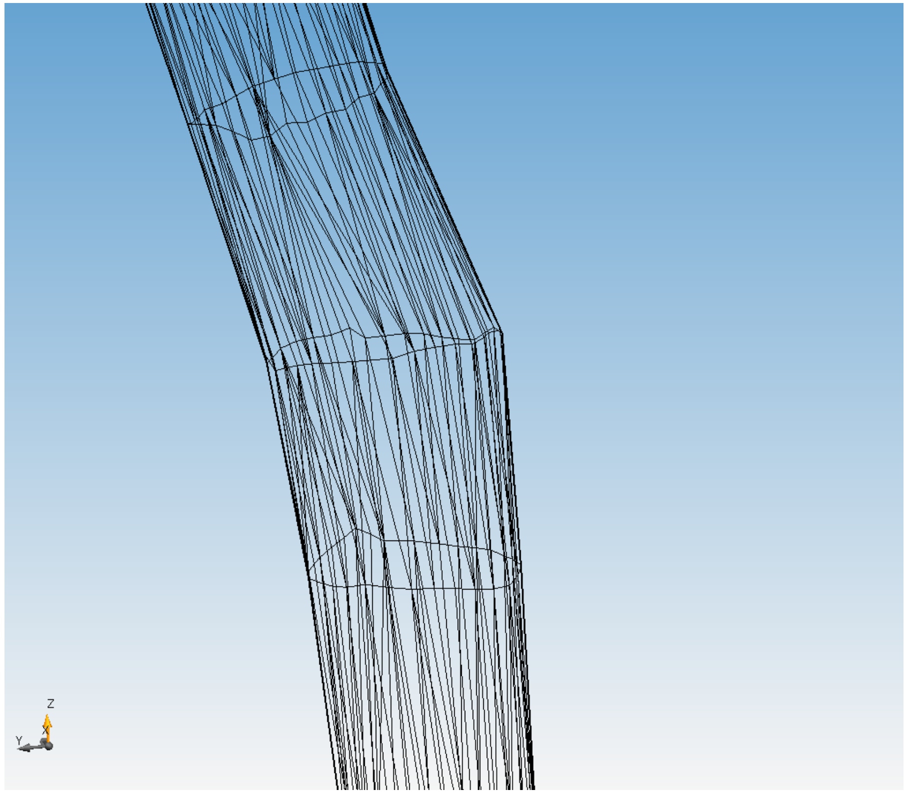
$$u = (P_{n-1,u}, P_{n,b})$$

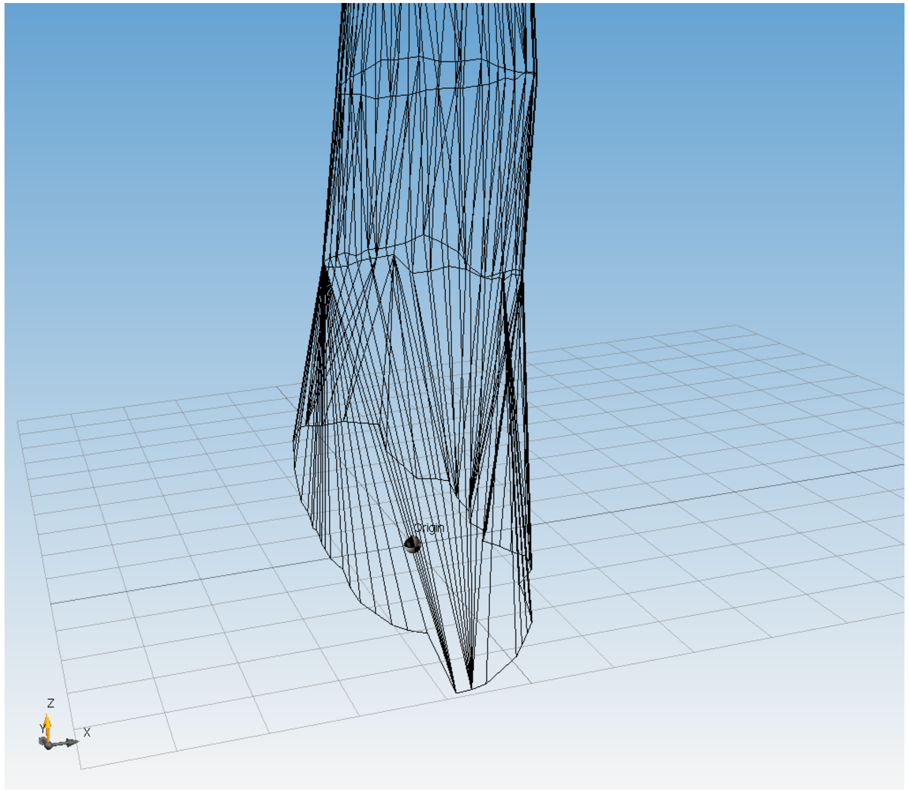
Second angle is defined respectively

Application of surface matching method also known as gift wrapping algorithm

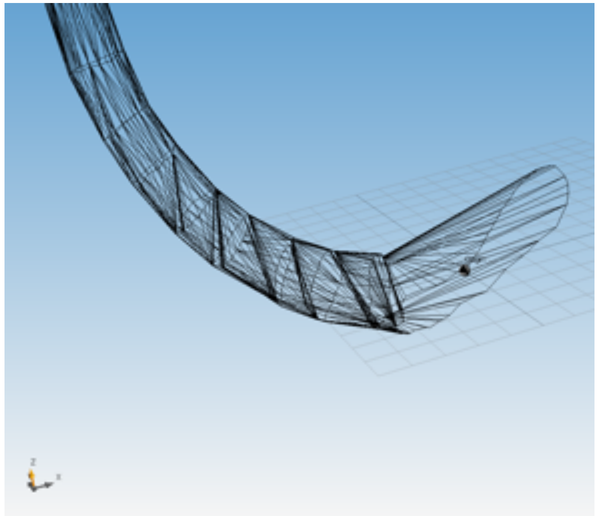
+ solves problems of unequally horizontal points
+ still relatively easy and computationally not demanding

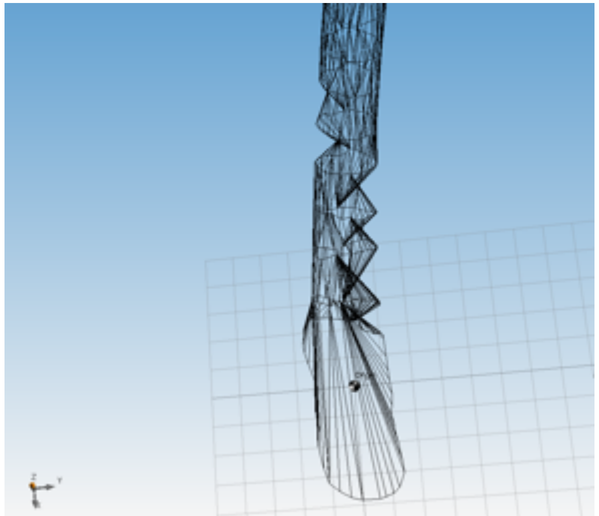
- Fails to work on bended trees and extreme unexpected vertical elevations.

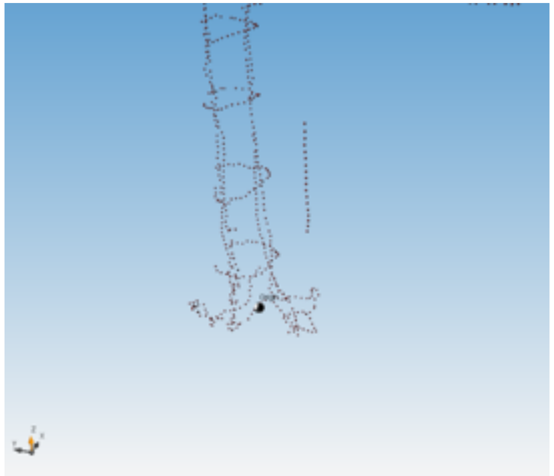


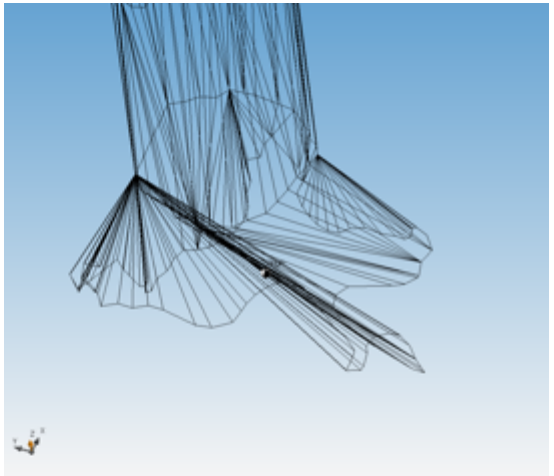


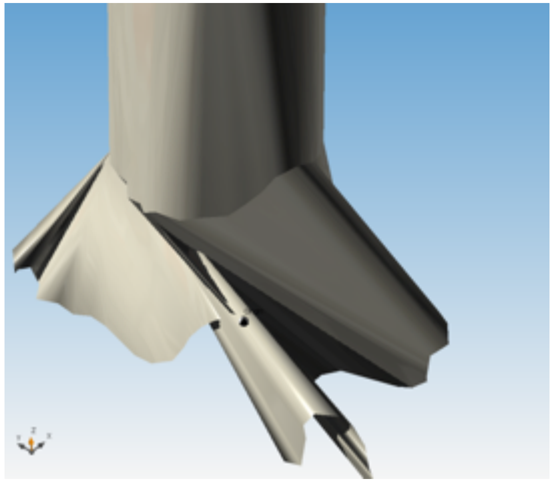
Minimum angle











Minimum polar angle

Minimum angle of

$$P_n = \min(\tan(r,u), \tan(q,v))$$

Where:
r is a plane given by equation :

$$(x,y,z)=(x_0 + \lambda u_1 + \mu v_1, y_0 + \lambda u_2 + \mu v_2, z_0 + \lambda u_3 + \mu v_3)$$

Where

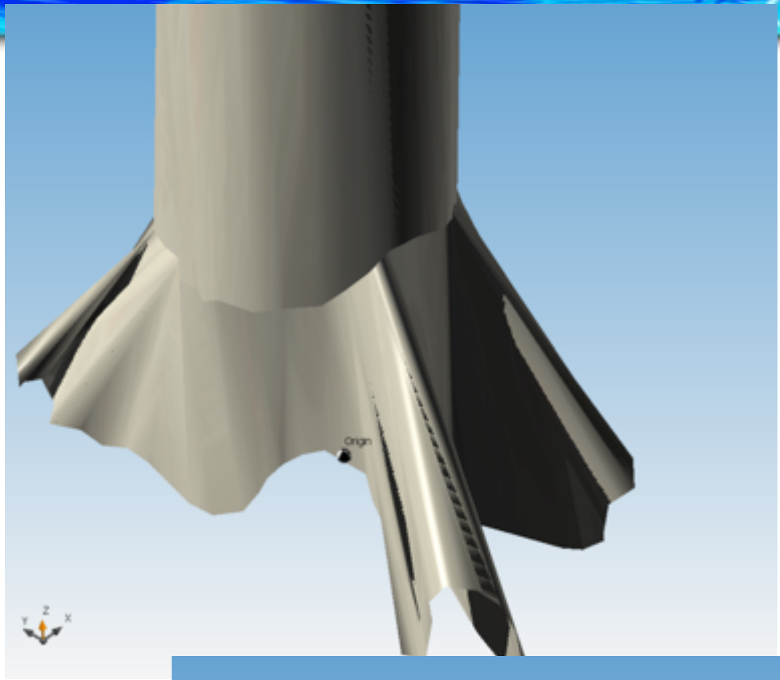
$C_u = (x_0, y_0, z_0)$ *//centroid of the upper point set*

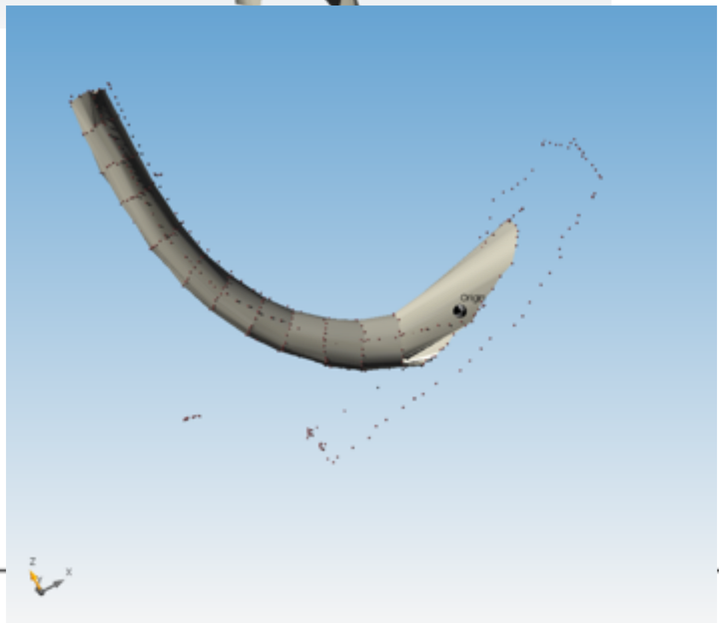
$u_1 = (C_u, P_{n-1}, u)$

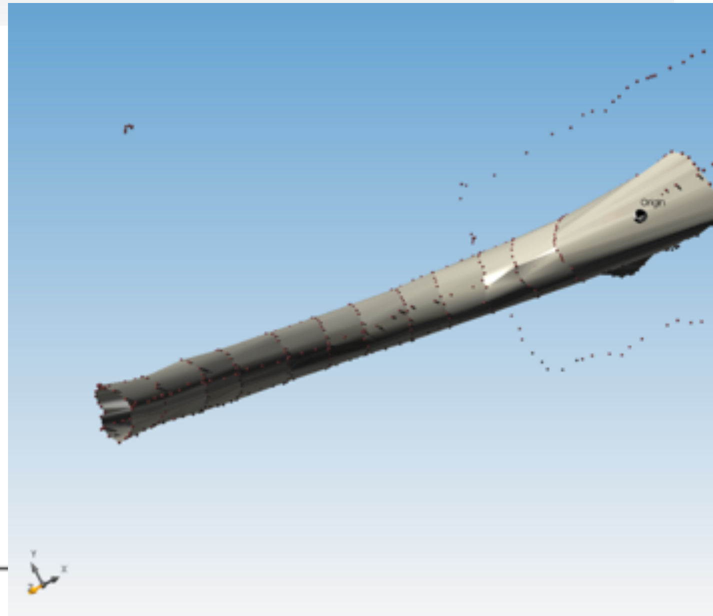
$v_1 = (C_u, C_b)$

$u = (C_u, P_n, b)$

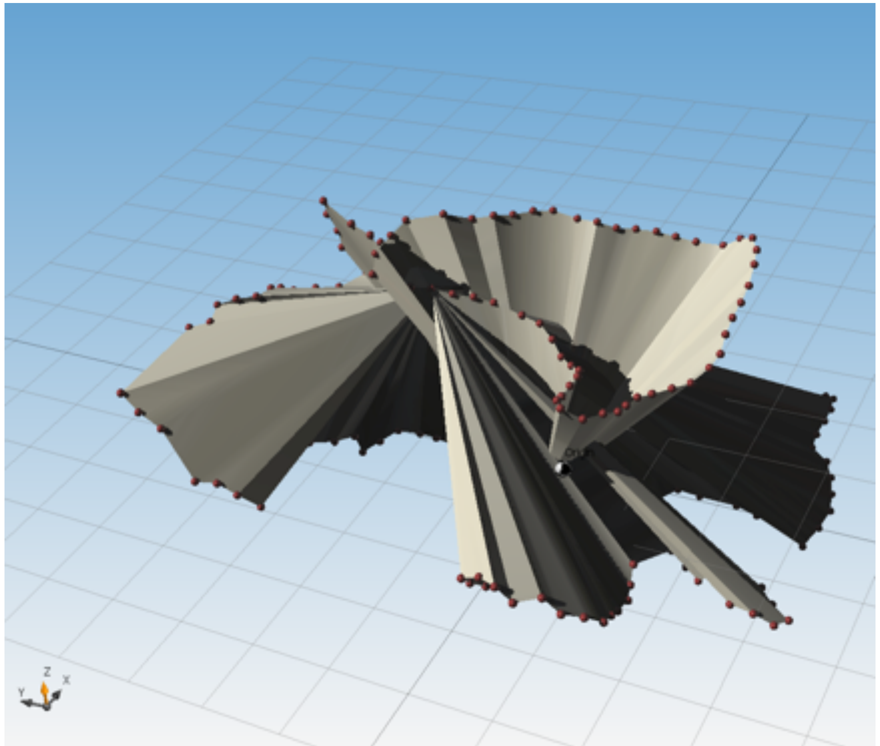
Second angle is defined respectively

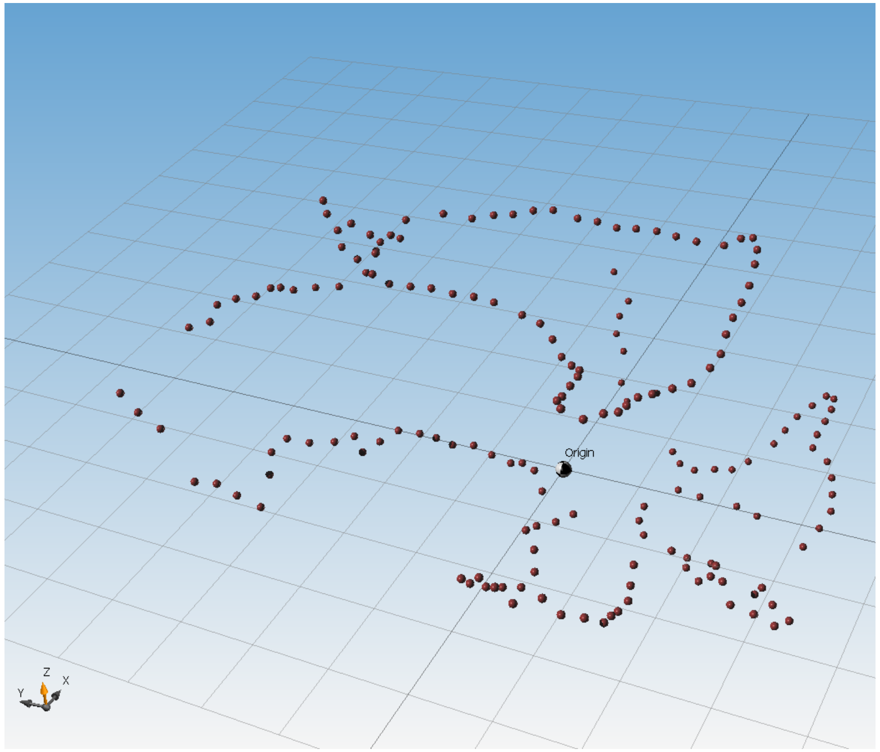






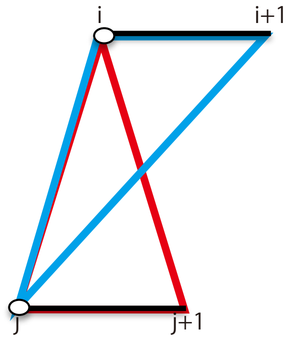
Problems





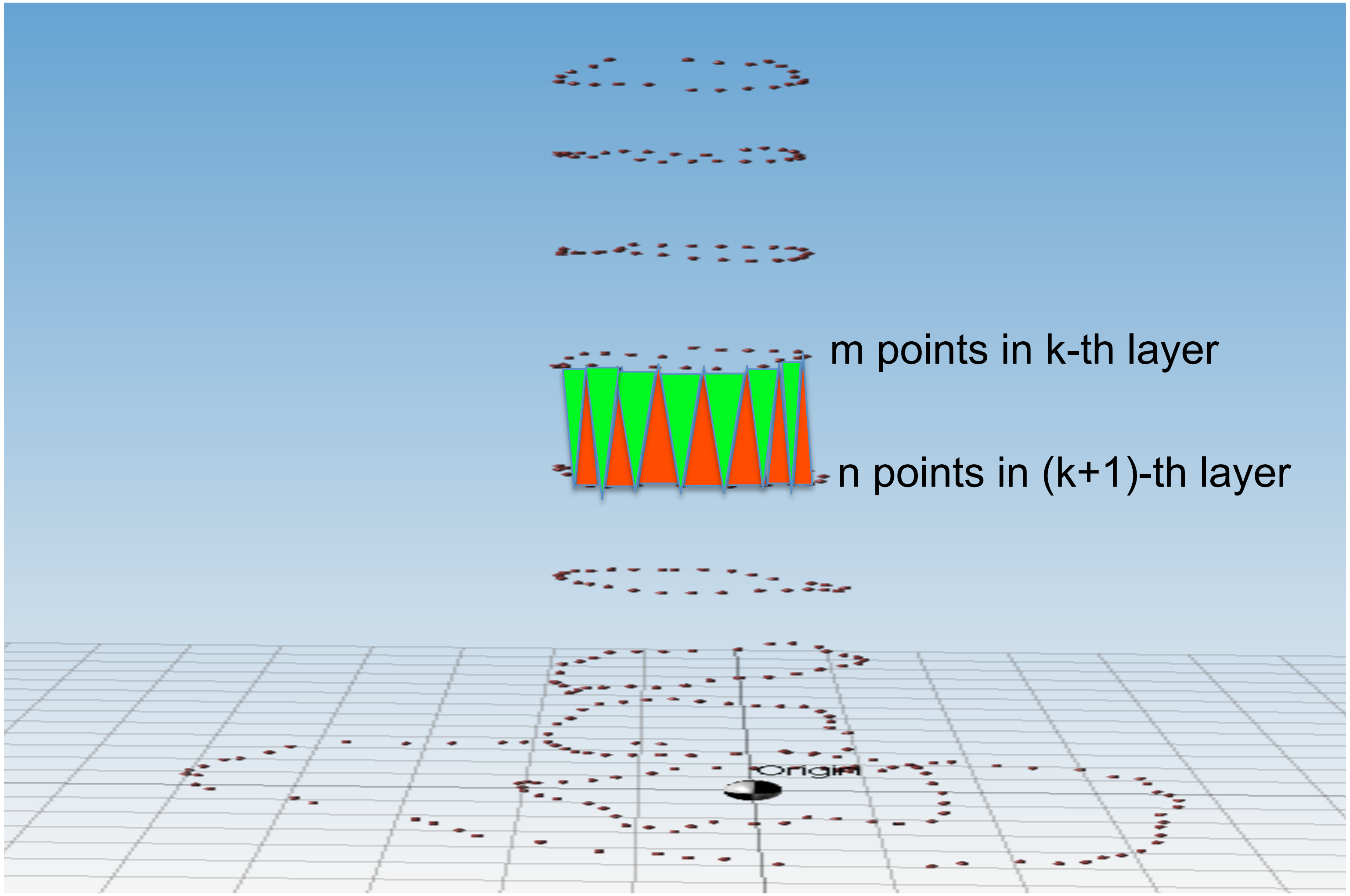
Integer Programming Formulation for Surface Reconstruction

Assume that there are *m* points in the *k*-th layer and *n* points in the (*k*+1)-st layer of the measured tree ($k = 1, 2, \dots K$) where *K* is the number of layers for one tree measured.

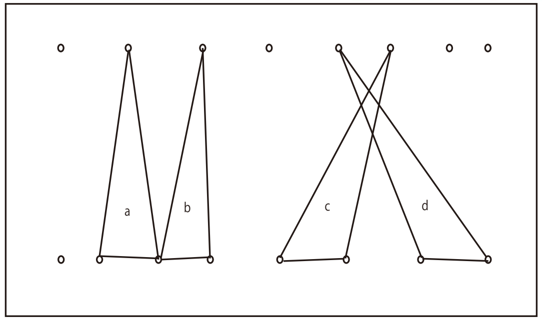


$$x^k_{i,j} = \begin{cases} 1 & \text{if the } i\text{-th point of the } k\text{-th layer is connected} \\ & \text{to } j\text{-th arc of the } (k{+}1)\text{st layer to form a triangle} \\ 0 & \text{otherwise} \end{cases}$$

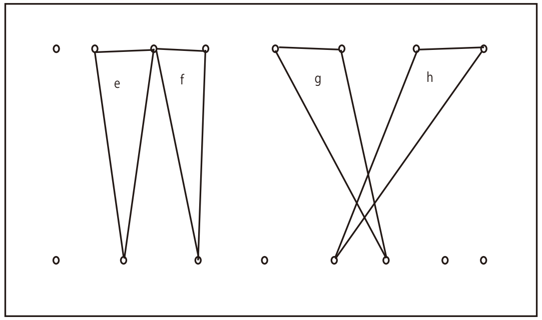
$$y^k_{j,i} = \begin{cases} 1 & \text{if the } i\text{-th arc of the } k\text{-th layer is connected} \\ & \text{to } j\text{-th point of the } (k{+}1)\text{st layer to form a triangle} \\ 0 & \text{otherwise} \end{cases}$$


m points in k-th layer
n points in (k+1)-th layer
Origin

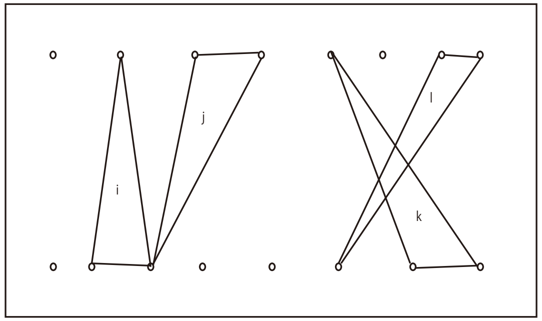
point-point

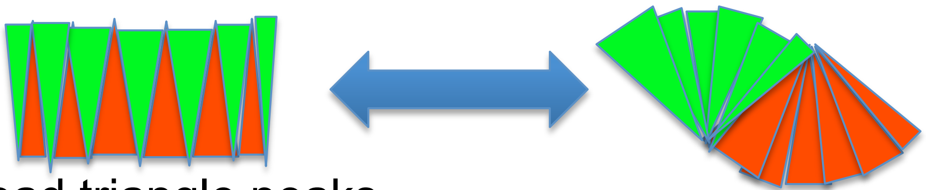


arc-arc



point-arc





Multiple Objective:

1. To maximize the number of points selected to form triangle peaks
   - This is to spread triangle peaks as much as possible
2. To minimize the total area of the surface constructed by triangles.

$$Z = \max_{\boldsymbol{X}_k, \boldsymbol{Y}_k} \{ p(\sum_{i=1}^{m} z^k{}_i + \sum_{j=1}^{n} w^k{}_j) - \sum_{i=1}^{m}\sum_{j=1}^{n}(c^k{}_{i,j} \cdot x^k{}_{i,j} + d^k{}_{j,i} \cdot y^k{}_{j,i}) \}$$

where

$p:$ weight for the total number big enough as to the area

$$x^k{}_{i,j} = \begin{cases} 1 & \text{if the } i\text{-th point of the } k\text{-th layer is connected} \\ & \text{to } j\text{-th arc of the } (k+1)\text{st layer to form a triangle} \\ 0 & \text{otherwise} \end{cases}$$

$$y^k{}_{j,i} = \begin{cases} 1 & \text{if the } i\text{-th arc of the } k\text{-th layer is connected} \\ & \text{to } j\text{-th point of the } (k+1)\text{st layer to form a triangle} \\ 0 & \text{otherwise} \end{cases}$$

$$z^k{}_i = \begin{cases} 1 & \text{if the } i\text{-th point of the } k\text{-th layer is selected to form a triangle} \\ 0 & \text{otherwise} \end{cases}$$

$$w^k{}_j = \begin{cases} 1 & \text{if the } j\text{-th point of the } (k+1)\text{st layer is selected to form a triangle} \\ 0 & \text{otherwise} \end{cases}$$

Constraints

Arc continuity constraints for x and y
each arc in the layer has to be exactly selected once.

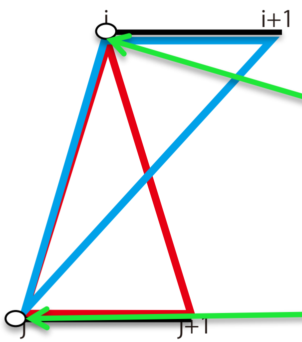
$$\sum_{i=1}^{m} x_{i,j}^{k} = 1, \qquad j = 1,2,..,n$$

$$\sum_{j=1}^{n} y_{j,i}^{k} = 1, \qquad i = 1,2,..,m$$

Linkage constrains between (z,w) to (x,y)

$$z^k{}_i = \begin{cases} 1 & \text{if the } i\text{-th point of the } k\text{-th layer is selected} \\ 0 & \text{otherwise} \end{cases}$$

$$w^k{}_j = \begin{cases} 1 & \text{if the } j\text{-th point of the } (k+1)\text{st layer is selected} \\ 0 & \text{otherwise} \end{cases}$$

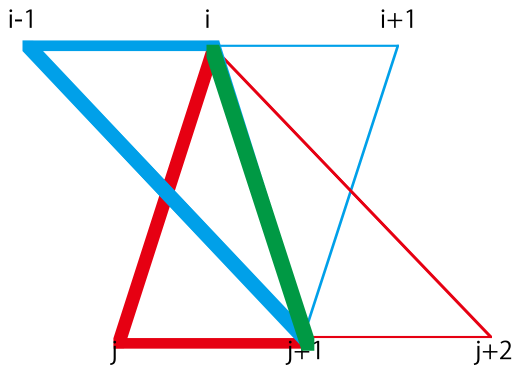


$$z_i^k \le \sum_{j=1}^{n} x_{i,j}^k, \qquad i = 1,2,..,m$$

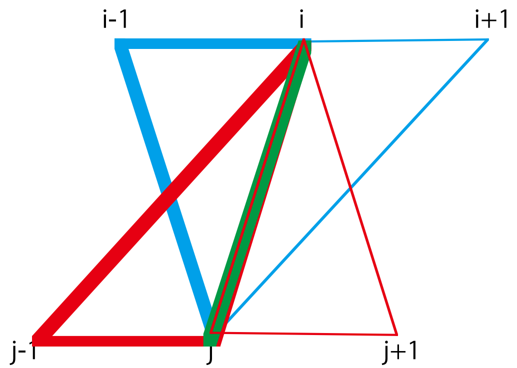
$$w_j^k \le \sum_{i=1}^{m} y_{i,j}^k, \qquad j = 1,2,..,n$$

Connectivity constraints

among triangles sharing the same boarder line

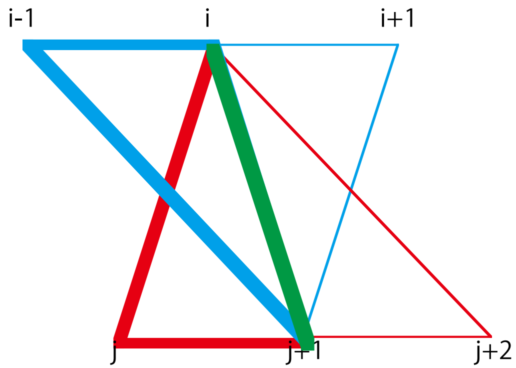


$$x^k_{i,j} + y^k_{j+1,i-1} = x^k_{i,j+1} + y^k_{j+1,i}$$

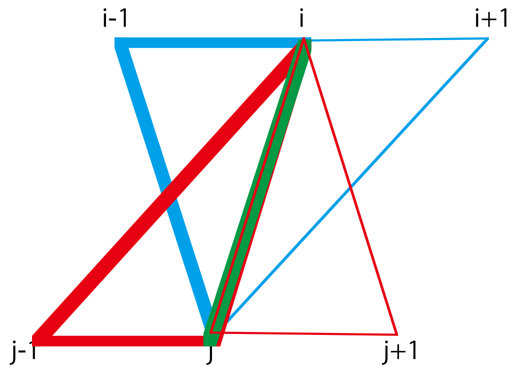


$$x^k_{i,j-1} + y^k_{j,i-1} = x^k_{i,j} + y^k_{j,i}$$

Overlap constraints to avoid local overlapped selection of triangles



$$x^k_{i,j} + y^k_{j+1,i-1} \leq 1$$
$$x^k_{i,j+1} + y^k_{j+1,i} \leq 1$$



$$x^k_{i,j} + y^k_{j,i} \leq 1$$
$$x^k_{i,j-1} + y^k_{j,i-1} \leq 1$$

IP Formulation

$$Z = \max_{\boldsymbol{X}_k, \boldsymbol{Y}_k} \{ p(\sum_{i=1}^{m} z^k{}_i + \sum_{j=1}^{n} w^k{}_j) - \sum_{i=1}^{m}\sum_{j=1}^{n} (c^k{}_{i,j} \cdot x^k{}_{i,j} + d^k{}_{j,i} \cdot y^k{}_{j,i}) \}$$

st

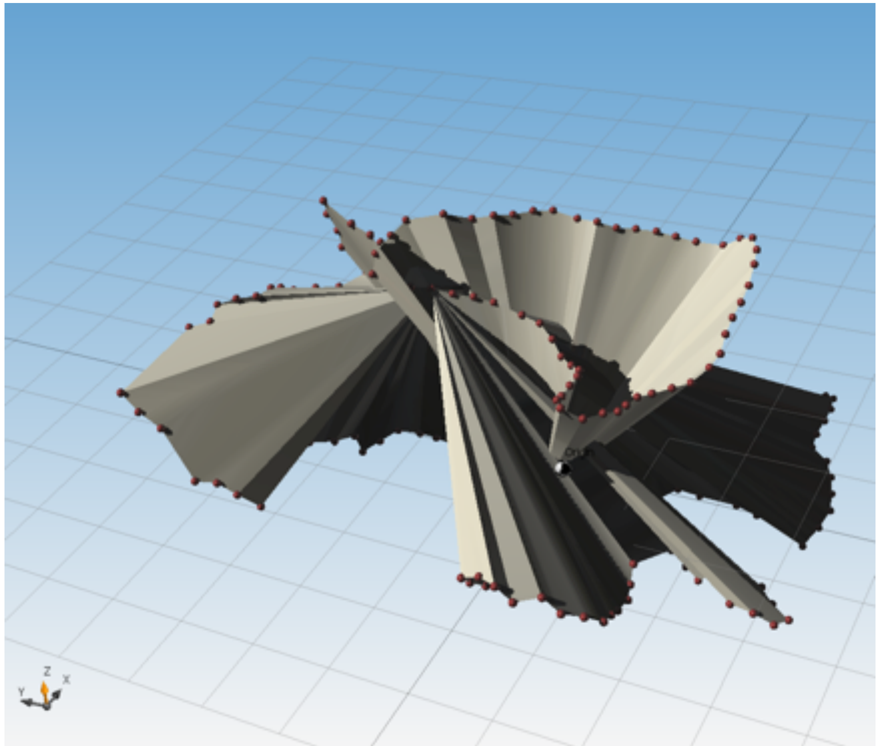
1. $\sum_{i=1}^{m} x_{i,j}^k = 1, \quad j = 1,2,..,n, \quad \sum_{j=1}^{n} y_{j,i}^k = 1, \quad i = 1,2,..,m$

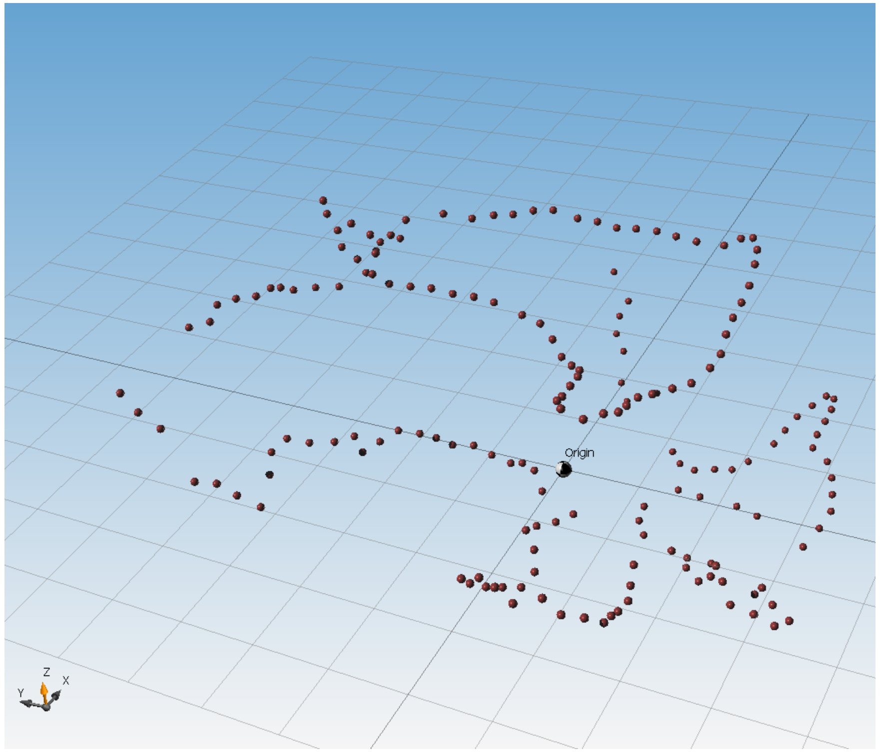
2. $z_i^k \le \sum_{j=1}^{n} x_{i,j}^k, \quad i = 1,2,..,m, \quad w_j^k \le \sum_{i=1}^{m} y_{i,j}^k, \quad j = 1,2,..,n$

3. $x_{i,j}^k + y_{j+1,i-1}^k = x_{i,j+1}^k + y_{j+1,i}^k, \quad x_{i,j-1}^k + y_{j,i-1}^k = x_{i,j}^k + y_{j,i}^k, \qquad \forall i,j$

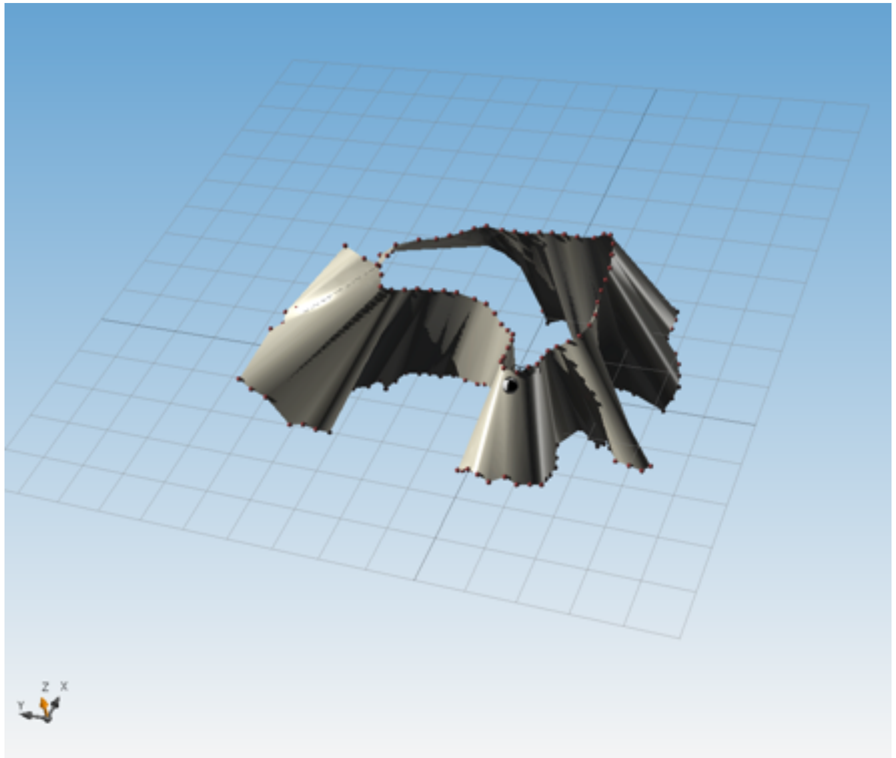
4. $x_{i,j}^k + y_{j+1,i-1}^k \le 1, \; x_{i,j+1}^k + y_{j+1,i}^k \le 1, \; x_{i,j}^k + y_{j,i}^k \le 1, \; x_{i,j-1}^k + y_{j,i-1}^k \le 1, \quad \forall i,j$

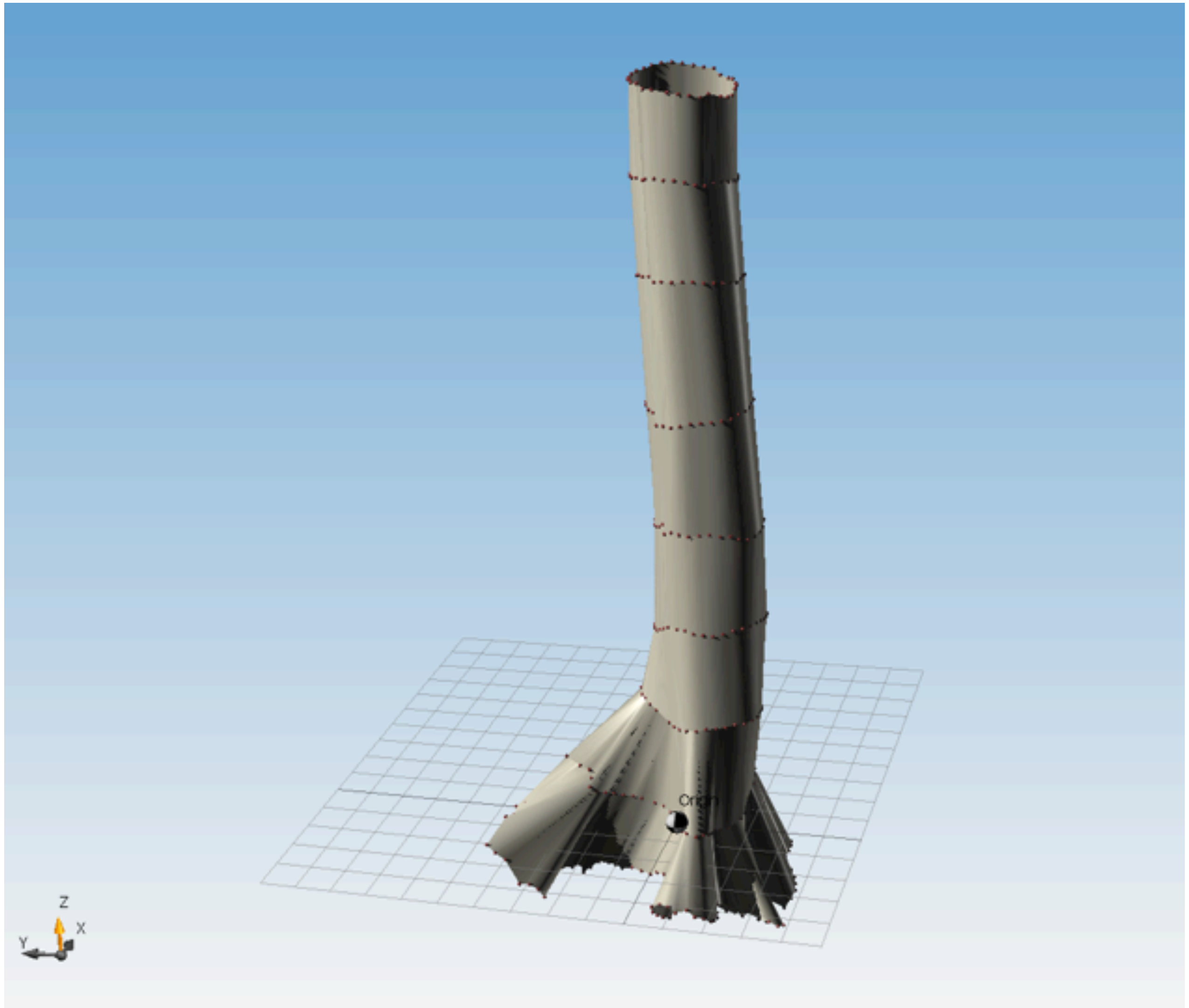
Complicated Case





Successful Smooth Surface!




Origin
Z
X
Y

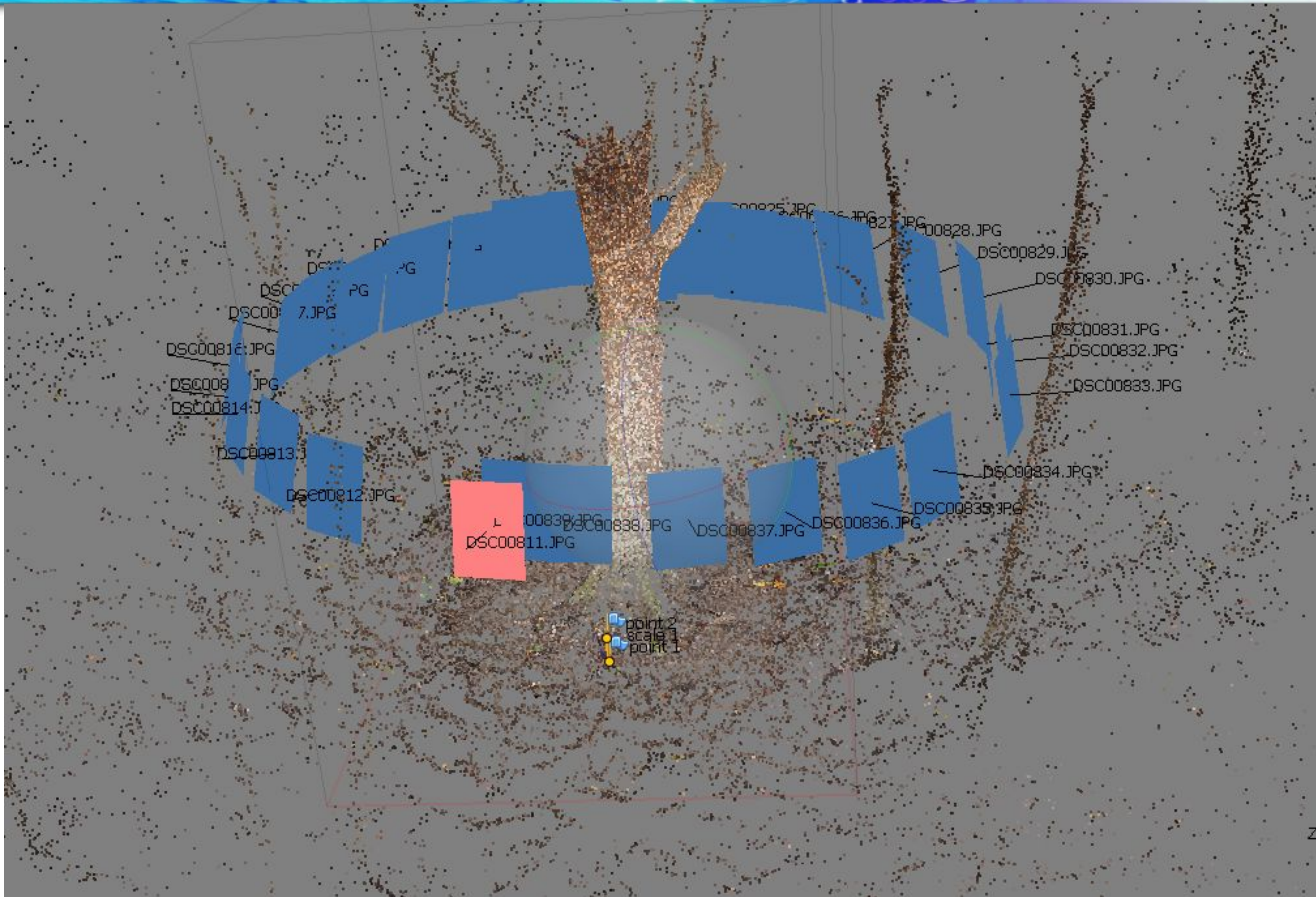
Possible usage and accuracy in individual species

Method	Fukugi	Itaji	Normal sugi	Bended sugi	Mangrove
I	o / Δ	Δ	o / Δ	x	x
II	o	Δ	o	x	x
III	o	Δ	o	o	x
Optimal	o	o	o	o	o

Other 3D measurements：Photogrametory

Terrestrial close-range photography: Single photo

Reconstructed point field:



rspective
point 2
scale 1
point 1
ces: 199999 vertices: 125798

3D animation of the reconstructed shape

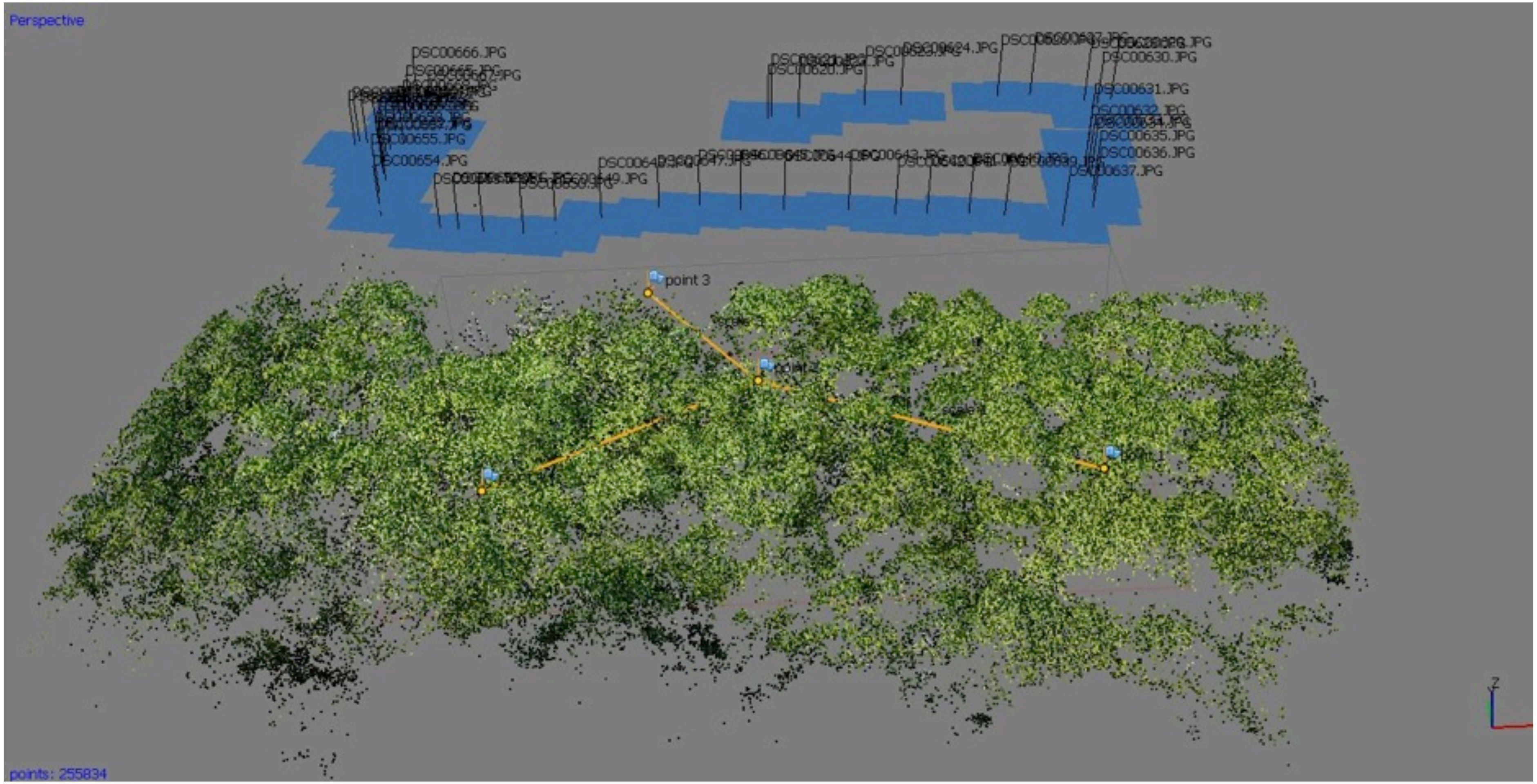
Measurements can be done with high precision and convinienently

DJI S800 spreading wings

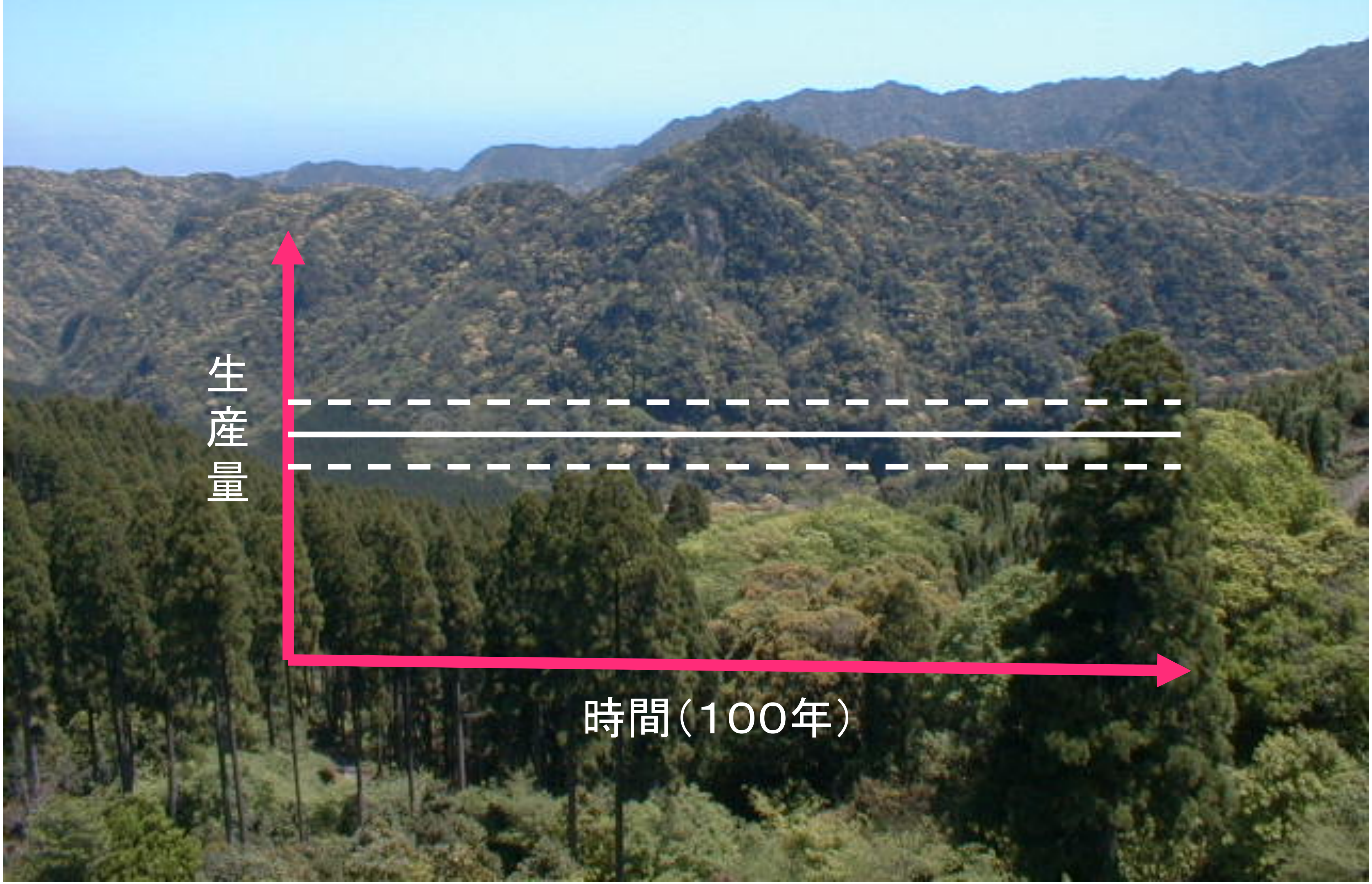
Flying the multicopter (top)

Ground field true data measurement (left)

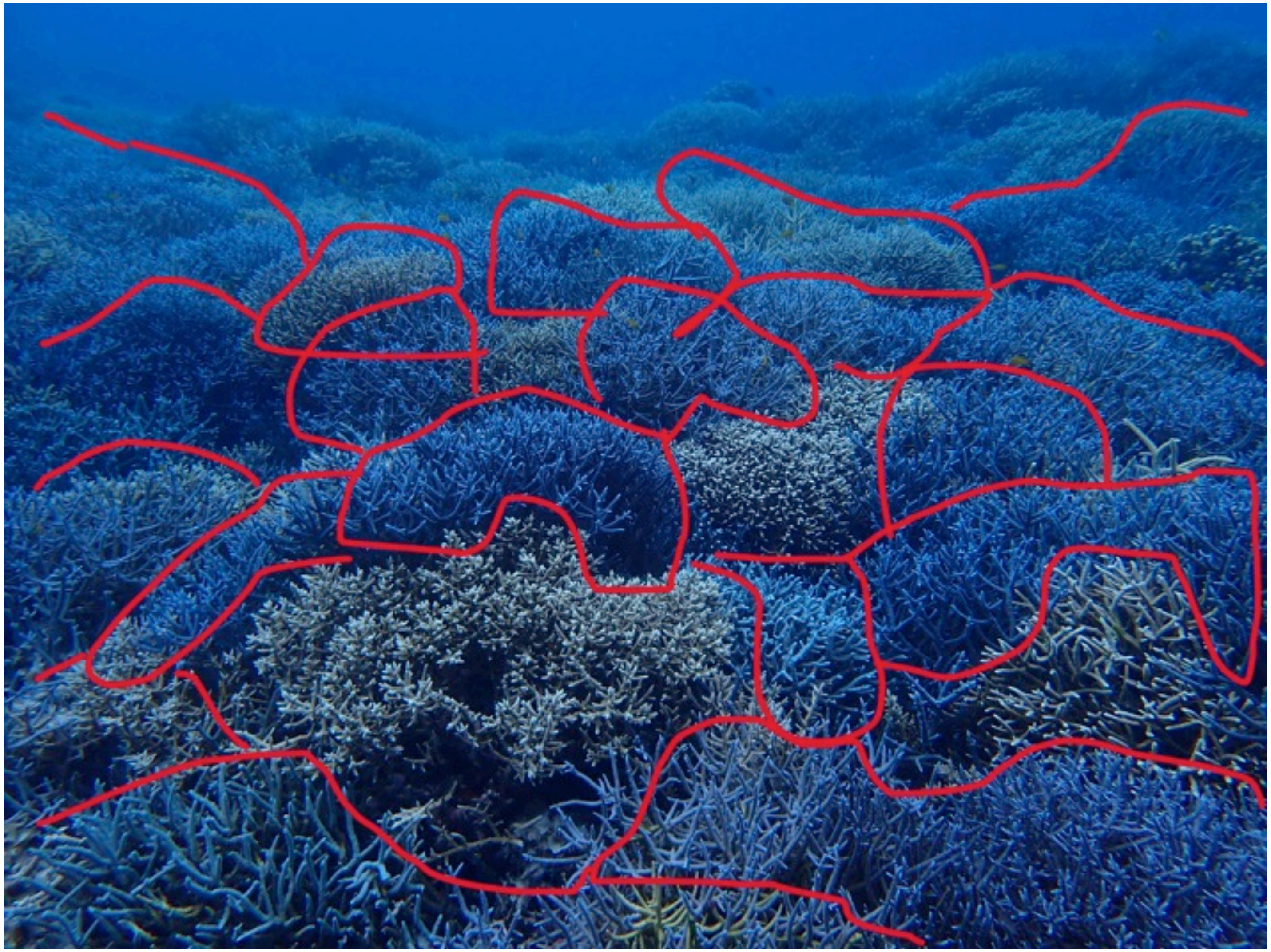
Point field reconstruction from individual photos:



森林レベルでの管理:広域的・長期的・持続的



森林？



伐採計画分野における初期の数理モデル

線型計画法Linear Programming (LP)

- TimberRAM (Navon 1971)
- MUSYC (Johnson and Jones 1979)
- FORPLAN (Johnson et al. 1986)

混合整数計画法Mixed Integer Programming (MIP)

- IRPM (Kirby et al. 1980)

Harvest, Road Network, Transportation

And many

数理計画法（LP, MIP & IP）の利点

定式の標準化

↓

OR 研究者によるソルバーの開発　CPLEX, GUROBI, SCIPなど

↓

応用分野の研究者は現実問題解決のための定式化に専念

主問題

$$\text{maximize}\, z = c_1x_1 + c_2x_2 + \cdots + c_nx_n$$

$$\textit{subject to}$$

$$a_{1,1}x_1 + a_{1,2}x_2 + \cdot\cdot + a_{1,n}x_n \le b_1$$

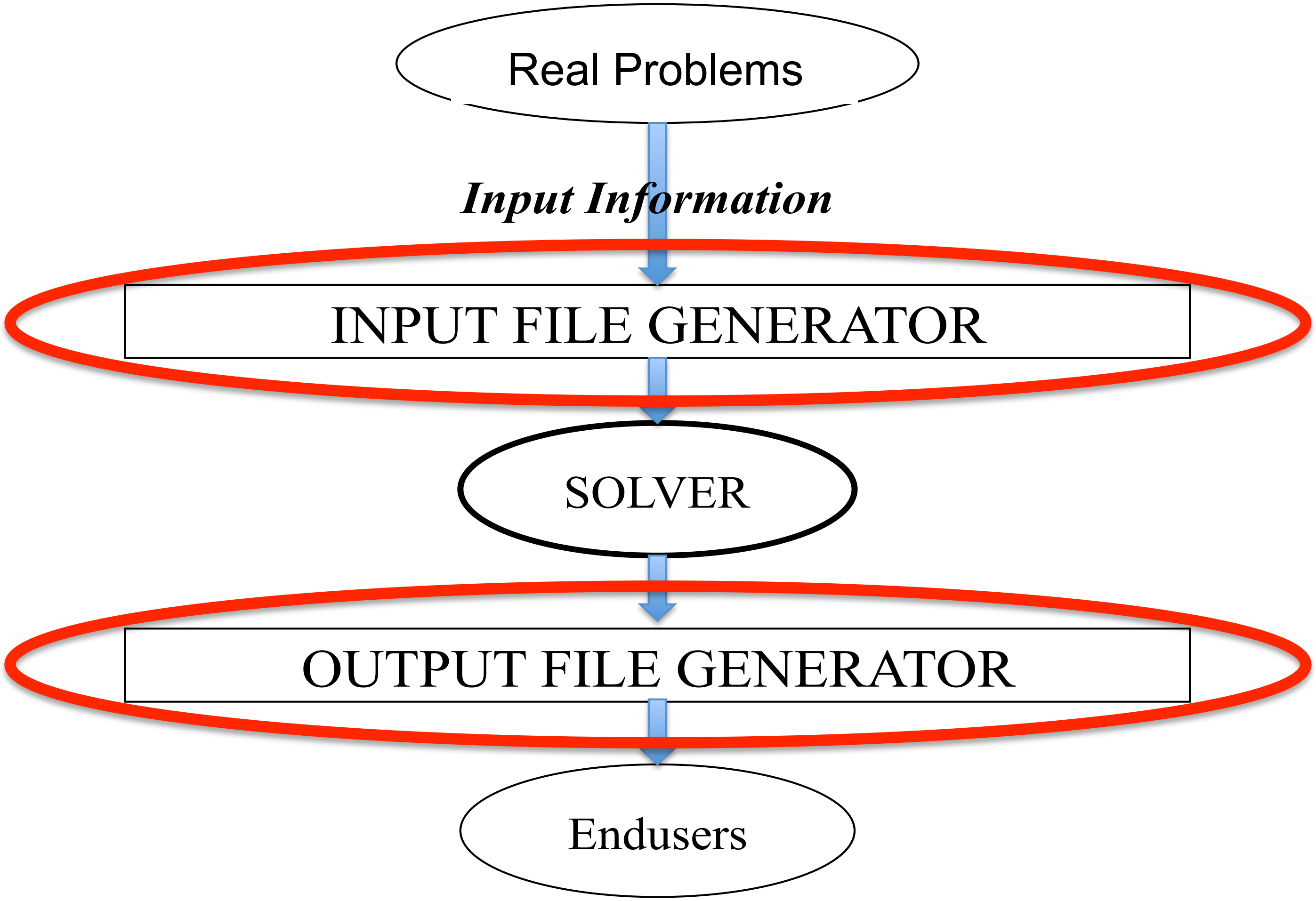
$$a_{2,1}x_1 + a_{2,2}x_2 + \cdot\cdot + a_{2,n}x_n \le b_2$$

$$\cdot$$

$$\cdot$$

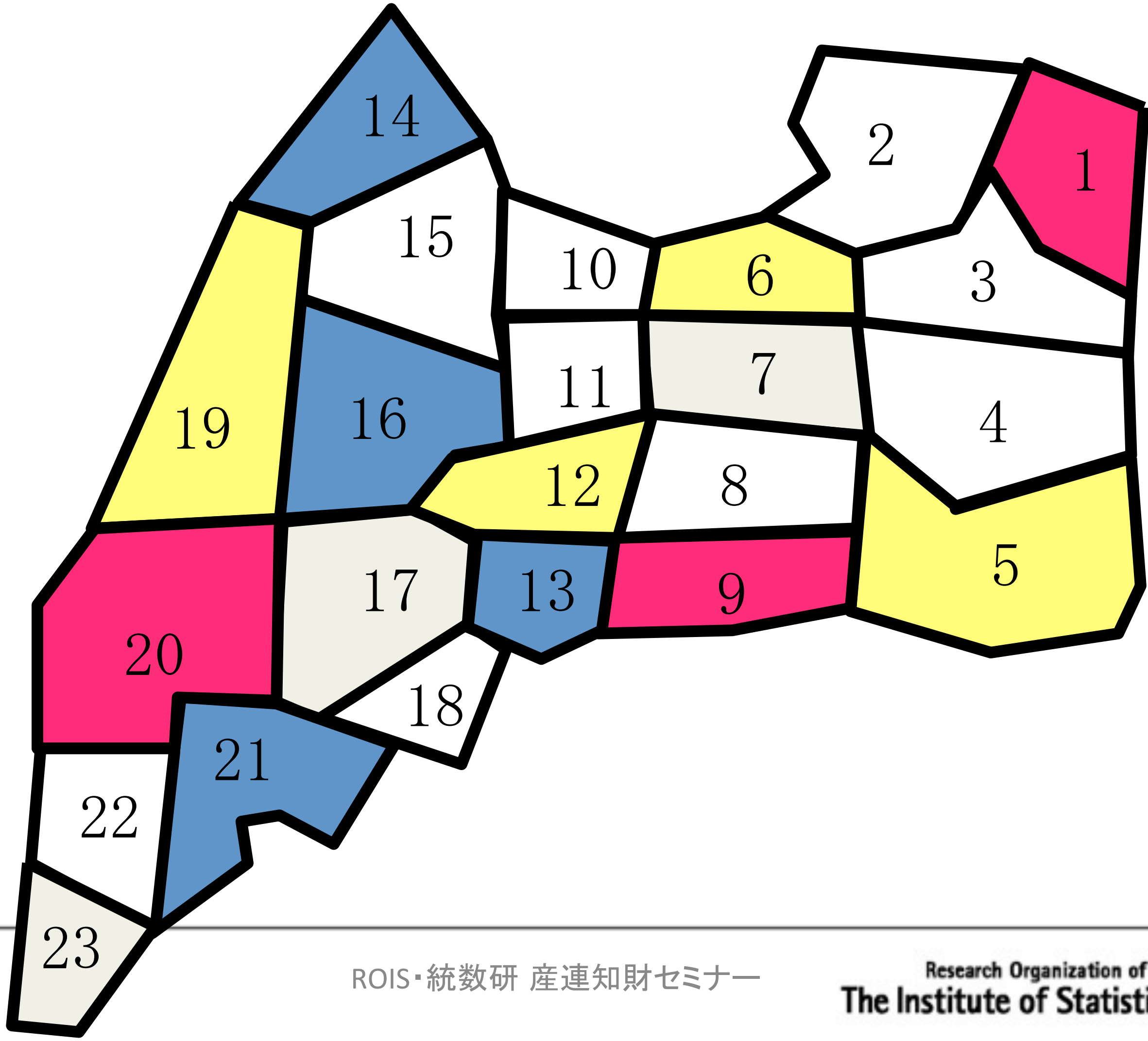
$$a_{m,1}x_1 + a_{m,2}x_2 + \cdot\cdot + a_{m,n}x_n \le b_m$$

$$x_1 \ge 0, x_2 \ge 0, \cdots, x_n \ge 0,$$


Real Problems
Input Information
INPUT FILE GENERATOR
SOLVER
OUTPUT FILE GENERATOR
Endusers

モデリングの概念

複数林分の森林レベル
制御時期，場所，強度：時空間を考慮



最適解探求には最適化の枠組みで定式化

基本構造の構築

- 決定変数を決める
- 必要な係数を推定する（予測値など）
- 最適化の目的を決める
- 制約条件を列挙する
  - 土地利用に関わる制約
  - 生産量に関わる制約
  - など

Techniques for Prescribing Optimal Timber Harvest and Investment Under Different Objectives— Discussion and Synthesis

BY

K. NORMAN JOHNSON

H. LYNN SCHEURMAN

Forest Science Monograph 18

1977

定式化：LPによる対応　70年代

- Johnson and Scheurman (1977) For.Sci.
- 伐採計画の定式化：植林ー伐採

- Model I　&　Model II　の開発：FORPLAN

決定変数: 施業

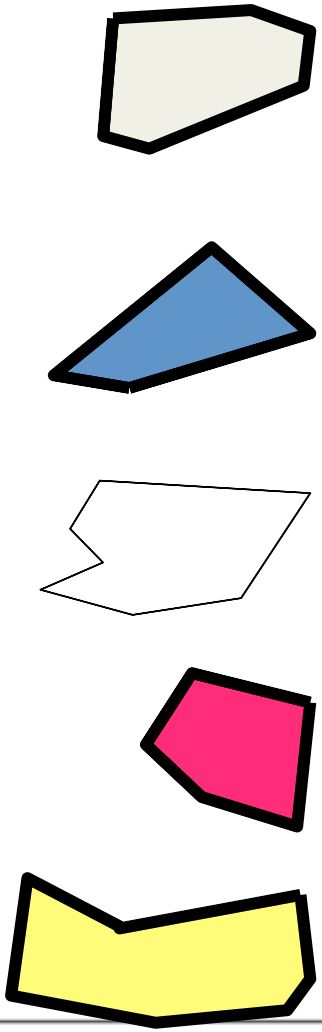
各期にどのグループからどれだけ伐採するのか？

	1st	2nd	3rd	4th	5th
x_{i5}	-	-	-	-	X
x_{i4}	-	-	-	X	-
x_{i3}	-	-	X	-	-
x_{i2}	-	X	-	-	-
x_{i1}	X	-	-	-	-

Period

$$\sum_h x_{ih} \leq 1$$

各分類でいつ，どれだけ？



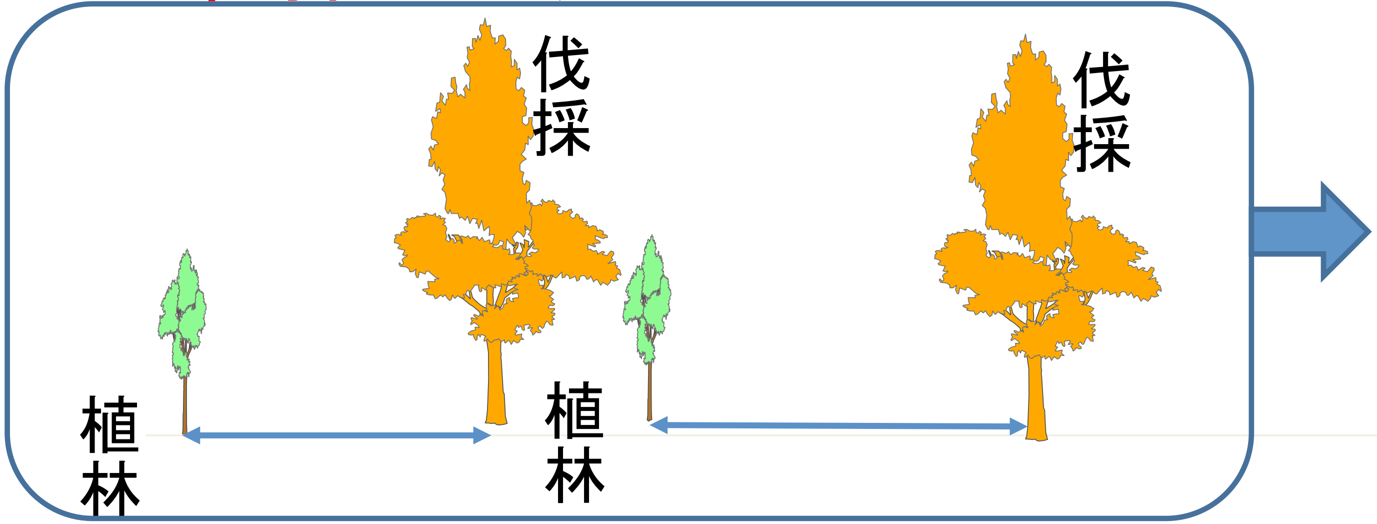
	1st	2nd	3rd	4th	5th
	10%	30%	10%	20%	30%
	5%	25%	35%	25%	10%
	30%	0%	30%	30%	10%
	50%	5%	10%	10%	25%
	10%	20%	30%	40%	0%

Time

森林管理モデル

Model I

伐採に対する静的施業Static Treatment



複数回の伐採
1つ施業：決定変数

Time

静的施業(Static Treatment)

Table 1: Example of treatments

最小伐期: 10期計画で6期

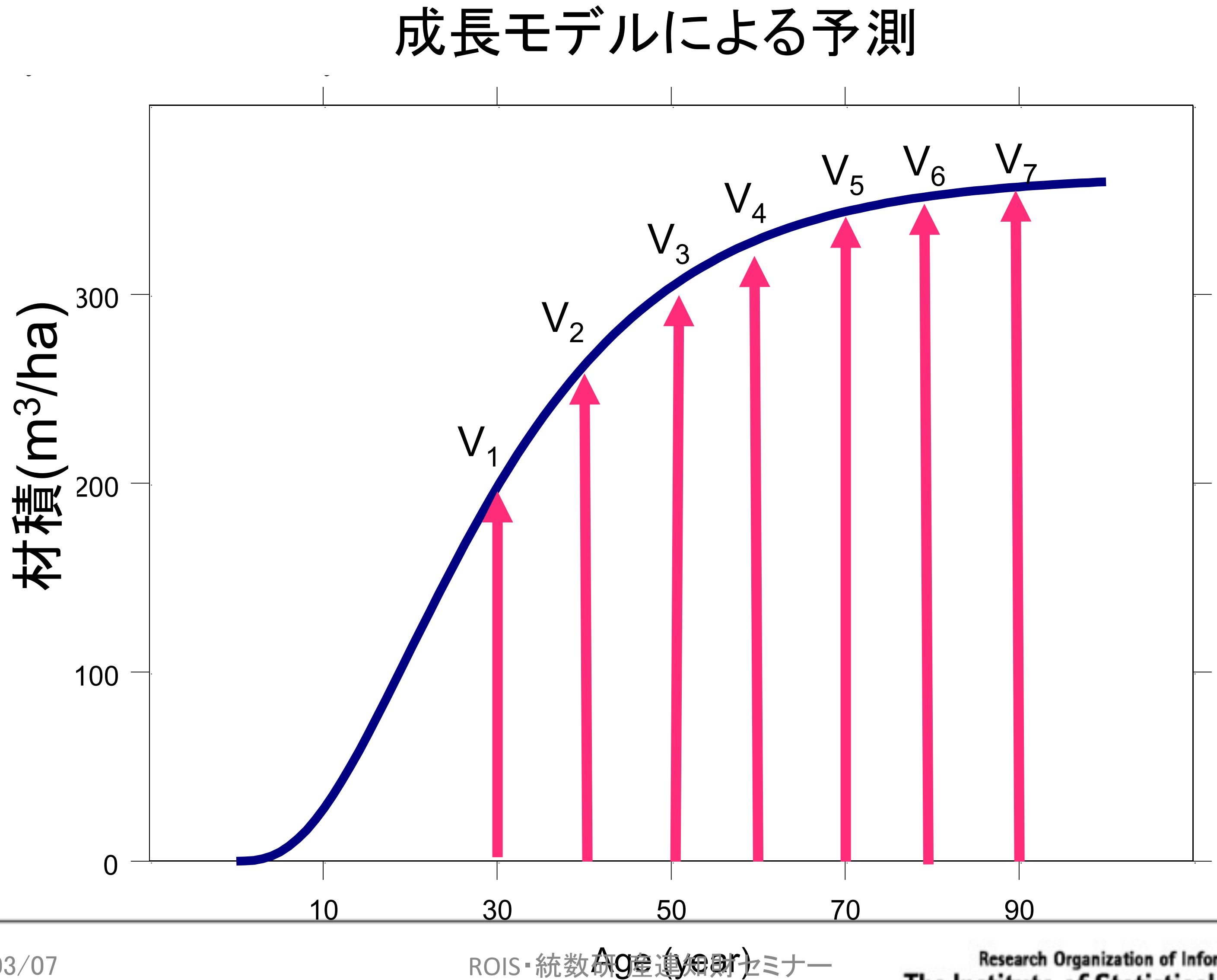
施業案h

Treatment No.	Decision Variable	Coefficient	1	2	3	4	5	6	7	8	9	10
			期間									
1	$x_{i,1}$	$c_{i,1}$	X	0	0	0	0	0	0	0	0	0
2	$x_{i,2}$	$c_{i,2}$	X	0	0	0	0	0	X	0	0	0
3	$x_{i,3}$	$c_{i,3}$	X	0	0	0	0	0	0	X	0	0
4	$x_{i,4}$	$c_{i,4}$	X	0	0	0	0	0	0	0	X	0
5	$x_{i,5}$	$c_{i,5}$	X	0	0	0	0	0	0	0	0	X
6	$x_{i,6}$	$c_{i,6}$	0	X	0	0	0	0	0	0	0	0
7	$x_{i,7}$	$c_{i,7}$	0	X	0	0	0	0	0	X	0	0
8	$x_{i,8}$	$c_{i,8}$	0	X	0	0	0	0	0	0	X	0
9	$x_{i,9}$	$c_{i,9}$	0	X	0	0	0	0	0	0	0	X
10	$x_{i,10}$	$c_{i,10}$	0	0	X	0	0	0	0	0	0	0
11	$x_{i,11}$	$c_{i,11}$	0	0	X	0	0	0	0	0	X	0
12	$x_{i,12}$	$c_{i,12}$	0	0	X	0	0	0	0	0	0	X
13	$x_{i,13}$	$c_{i,13}$	0	0	0	X	0	0	0	0	0	0
14	$x_{i,14}$	$c_{i,14}$	0	0	0	X	0	0	0	0	0	X
15	$x_{i,15}$	$c_{i,15}$	0	0	0	0	X	0	0	0	0	0
16	$x_{i,16}$	$c_{i,16}$	0	0	0	0	0	X	0	0	0	0
17	$x_{i,17}$	$c_{i,17}$	0	0	0	0	0	0	X	0	0	0
18	$x_{i,18}$	$c_{i,18}$	0	0	0	0	0	0	0	X	0	0
19	$x_{i,19}$	$c_{i,19}$	0	0	0	0	0	0	0	0	X	0
20	$x_{i,20}$	$c_{i,20}$	0	0	0	0	0	0	0	0	0	X

Note: X denotes harvesting while 0 denotes no harvesting

x: 伐採を示す

伐採量の推定


成長モデルによる予測
材積(m3/ha)
Age (year)
0
100
200
300
10
30
50
70
90
V1
V2
V3
V4
V5
V6
V7

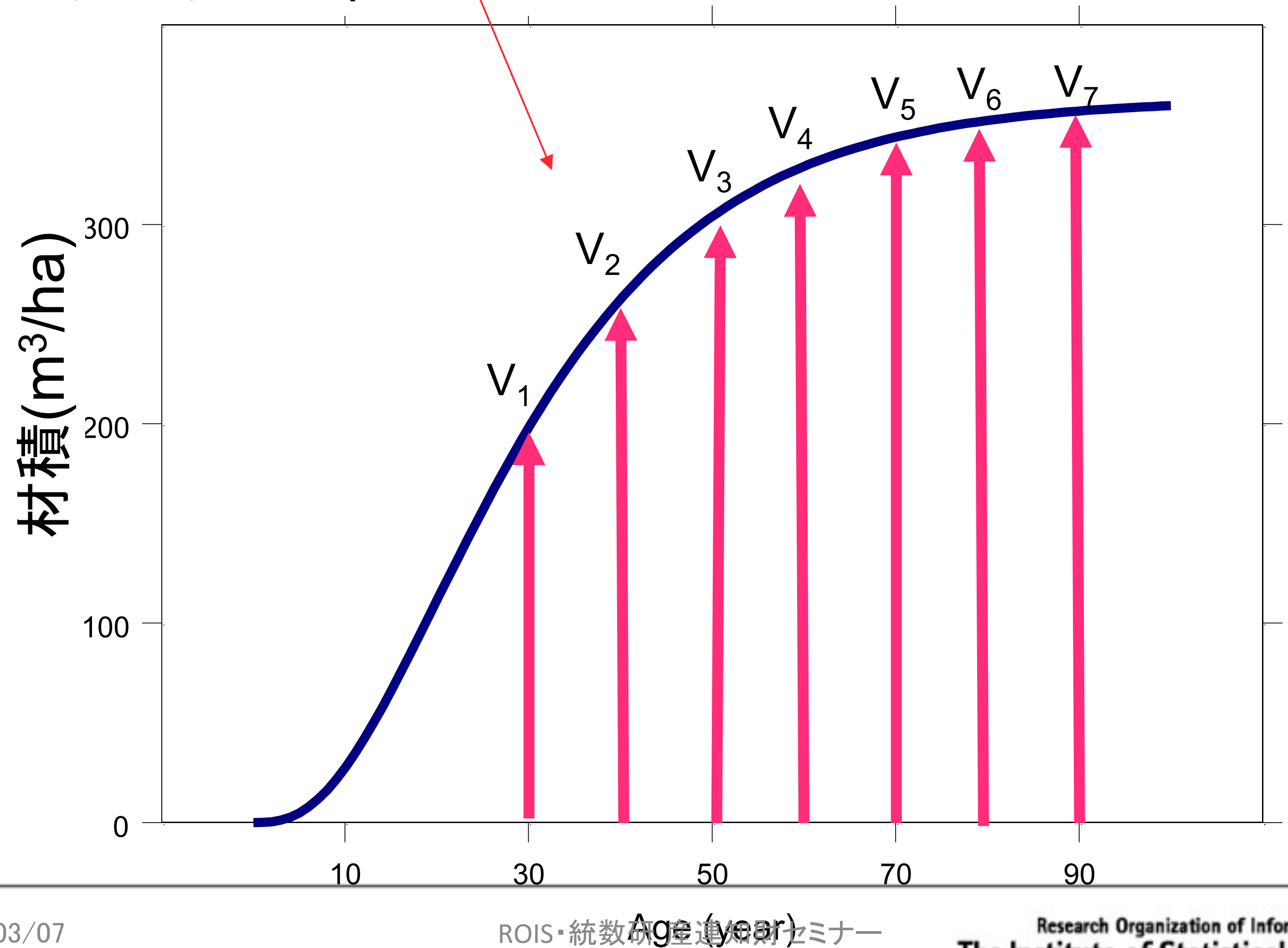
各期の伐採量

施業案 h	$v_{ih}^{(1)}$	$v_{ih}^{(2)}$	$v_{ih}^{(3)}$	$v_{ih}^{(4)}$	計画期間		$v_{ih}^{(7)}$	$v_{ih}^{(8)}$	$v_{ih}^{(9)}$	$v_{ih}^{(10)}$
	1	2	3	4	5	6	7	8	9	10
1	V_1	0	0	0	0	0	0	0	0	0
2	V_1	0	0	0	0	0	V_1	0	0	0
3	V_1	0	0	0	0	0	0	V_2	0	0
4	V_1	0	0	0	0	0	0	0	V_3	0
5	V_1	0	0	0	0	0	0	0	0	V_4
6	0	V_2	0	0	0	0	0	0	0	0
7	0	V_2	0	0	0	0	0	V_1	0	0
8	0	V_2	0	0	0	0	0	0	V_2	0
9	0	V_2	0	0	0	0	0	0	0	V_3
10	0	0	V_3	0	0	0	0	0	0	0
11	0	0	V_3	0	0	0	0	0	V_1	0
12	0	0	V_3	0	0	0	0	0	0	V_2
13	0	0	0	V_4	0	0	0	0	0	0
14	0	0	0	V_4	0	0	0	0	0	6
15	0	0	0	0	V_5	0	0	0	0	0
16	0	0	0	0	0	V_6	0	0	0	0
17	0	0	0	0	0	0	V_7	0	0	0
18	0	0	0	0	0	0	0	V_8	0	0
19	0	0	0	0	0	0	0	0	V_9	0
20	0	0	0	0	0	0	0	0	0	V_{10}
	$_h\,v_{ih}^{(1)}x_{ih}$	$_h\,v_{ih}^{(2)}x_{ih}$	$_h\,v_{ih}^{(3)}x_{ih}$	$_h\,v_{ih}^{(4)}x_{ih}$	$_h\,v_{ih}^{(5)}x_{ih}$	$_h\,v_{ih}^{(6)}x_{ih}$	$_h\,v_{ih}^{(7)}x_{ih}$	$_h\,v_{ih}^{(8)}x_{ih}$	$_h\,v_{ih}^{(9)}x_{ih}$	$_h\,v_{ih}^{(10)}x_{ih}$

伐採量の推定

$$c_{i,j} = \frac{1}{(1+r)^{d(p-1)}} \sum_{p} p v_{ih}^{(p)} x_{ih}$$

成長予測



線形計画法による定式化

基本構造Model Iによる伐採計画

$$Z = \max_{\mathbf{x}} \sum_{i=1}^{m} \sum_{h=1}^{H} c_{ih} \cdot x_{ih}$$

最適化ソフトウェア使用
Cplex, Gurobi, Scip
(Academic Free)

$st.$

$$x_{i1} + x_{i2} + \cdots + x_{iH} \leq 1, \quad \forall i$$

土地利用制約

$$\sum_{i=1}^{m} \sum_{h=1}^{H} v_{ih}^{(p)} \cdot x_{ih} = v_0, p = 1, \cdots, T$$

生産量制約
各期伐採量一定

$$\{x_{ih}\} \geq 0$$

0-1 整数計画法による定式化

離散型Model I

$$Z = \max_{\mathbf{x}} \sum_{i=1}^{m} \sum_{h=1}^{H} c_{ih} \cdot x_{ih}$$

$st.$

$$x_{i1} + x_{i2} + \cdots + x_{iH} \leq 1, \qquad \forall i$$

土地利用制約

$$(1-\alpha)v_0 \leq \sum_{i=1}^{m} \sum_{h=1}^{H} v_{ih}^{(p)} \cdot x_{ih} \leq (1+\alpha)v_0, \qquad p = 1, \cdots, T$$

生産量制約

$$\{x_{ih}\} = (0,1)$$

静的施業に対する決定変数0-1

決定変数: m forest units

$$x_{ij} = \begin{cases} 1 & \text{if } j\text{-th treatment is implemented at } i\text{-th unit} \\ 0 & \text{otherwise} \end{cases}$$

x_{i0} : no harvest activity

係数

c_{ij} : coefficient of x_{ij}

v_{it}^{p} : volume flow from x_{ij} at period p

Model I: Temporal Module

$$\max_{\{x_{ij}\}} Z = \sum_{i=1}^{m} \sum_{h=1}^{H} c_{ih} \times x_{ih}$$

$st.$

$$\sum_{h=0}^{H} x_{ih} = 1, \qquad \forall i$$

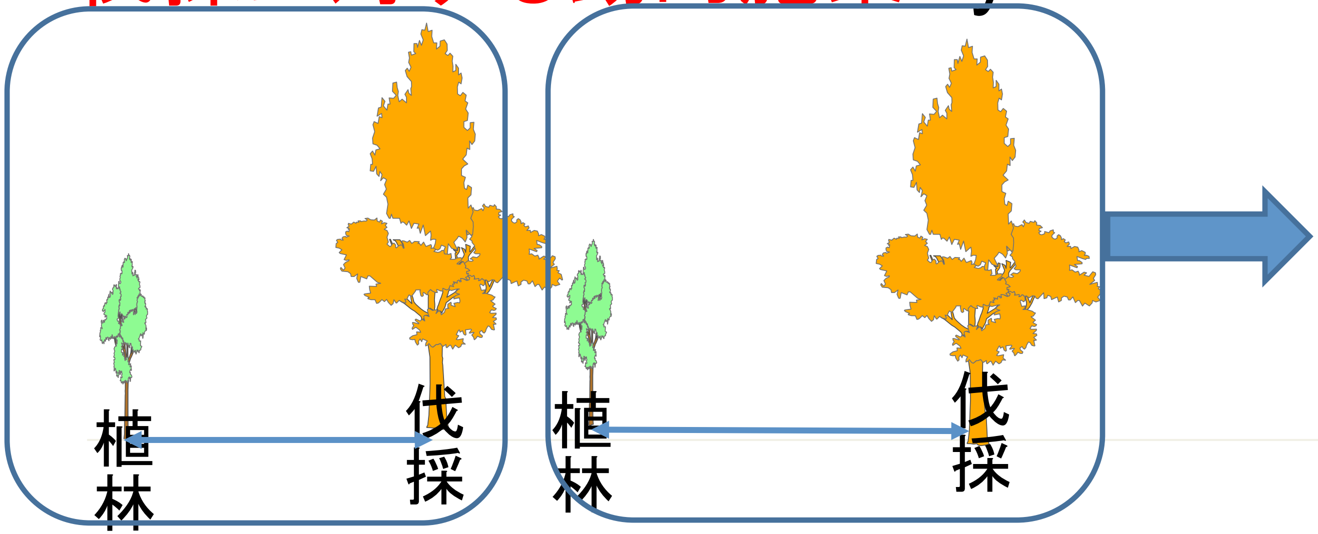
$$(1-\alpha)v_0 \le \sum_{i=1}^{m} \sum_{h=1}^{H} v_{ih}^{p} \times x_{ih} \le (1+\alpha)v_0, \qquad p = 1, \ldots, T$$

$$\{x_{ij}\} = (0, 1)$$

森林管理モデル

Model II

伐採に対する動的施業Dynamic Treatment

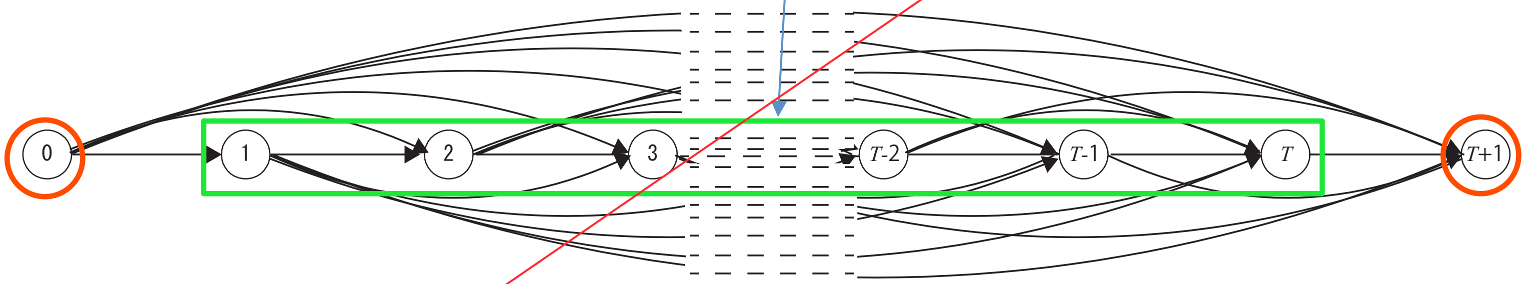


それぞれの伐採
1つ施業:決定変数

動的施業(Dynamic Treatment)

T期計画でk期の最小伐期

伐採の動態に対する動的ネットワーク: One-State & One-Stage



$x^i_{st} = 1$ if i-th unit is harvested at period t after plantaion at period s

$0 < s + k < t$

$$\sum_{\tau=0}^{\max(0,s-k)} x^i_{\tau s} = \sum_{t=\min(s+k,T+1)}^{T+1} x^i_{st}, \quad \forall i, s = 1, \ldots, T$$

$$\sum_{t=1}^{T+1} x^i_{0t} = 1, \qquad \forall i$$

動的施業に対する決定変数0-1

決定変数: m forest units

$$x^i_{st} = \begin{cases} 1 & \text{if } i\text{-th unit is harvested at period } t \text{ after plantaion at period } s \\ 0 & \text{otherwise} \end{cases}$$

係数

c^i_{st} : coefficient of x^i_{st}

v^i_{st} : volume flow from x^i_{st}

Model II: Temproral Module

$$\max Z = \sum_{i=1}^{m} \sum_{t=1}^{T+1} \sum_{s=0}^{\max(0,t-k)} c^i_{st} \times x^i_{st}$$

$st.$

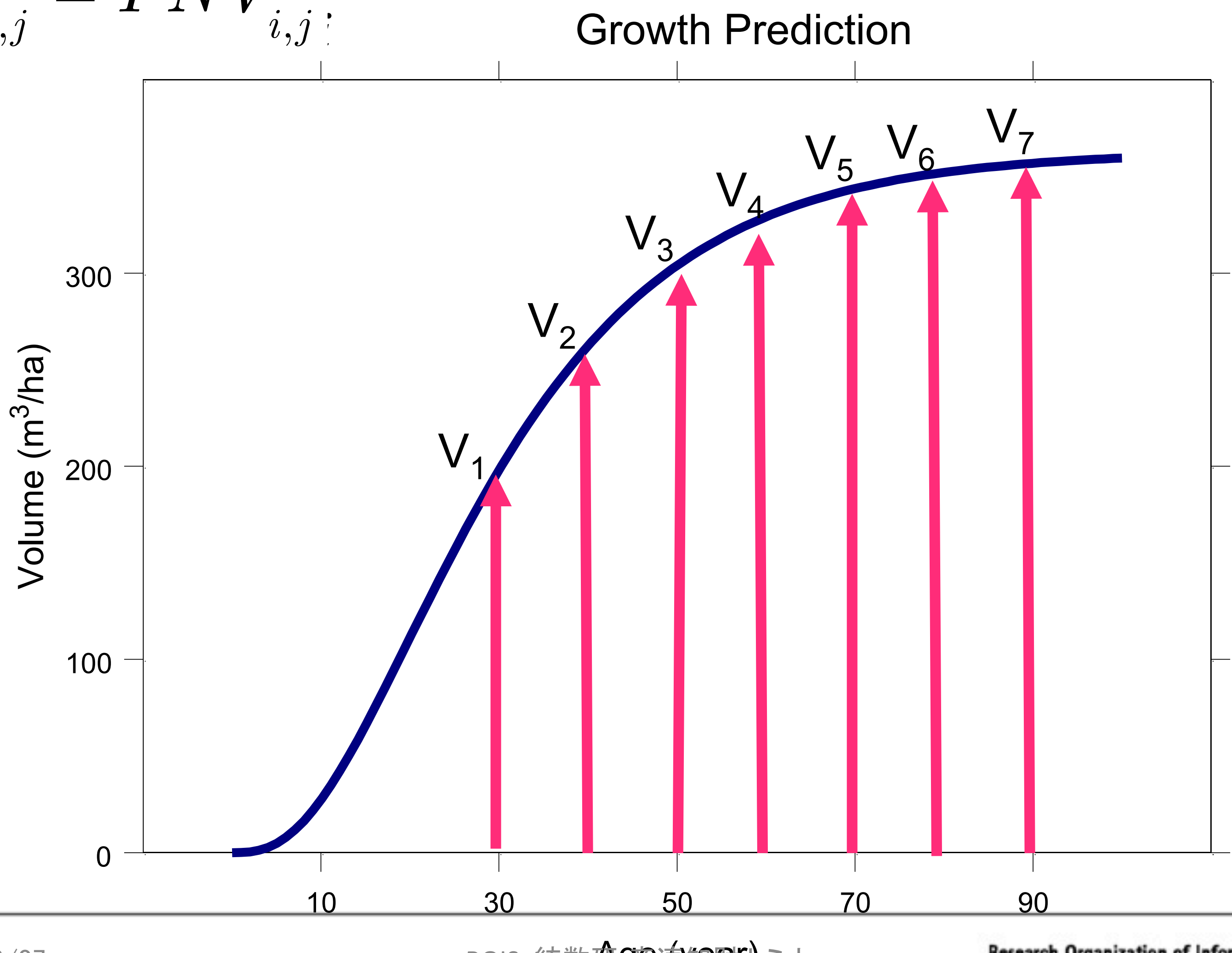
$$\sum_{t=0}^{\max(0,s-k)} x^i_{ts} = \sum_{t=\min(s+k,T+1)}^{T+1} x^i_{st}, \qquad \forall i, s = 1, \dots, T$$

$$\sum_{t=1}^{T+1} x^i_{0t} = 1, \qquad \forall i$$

$$(1-a) \times v_0 \le \sum_{i=1}^{m} \sum_{s=0}^{\max(0,t-k)} v^i_{st} \times x^i_{st} \le (1+a) \times v_0, \qquad t = 1, \dots, T$$

伐採量の推定

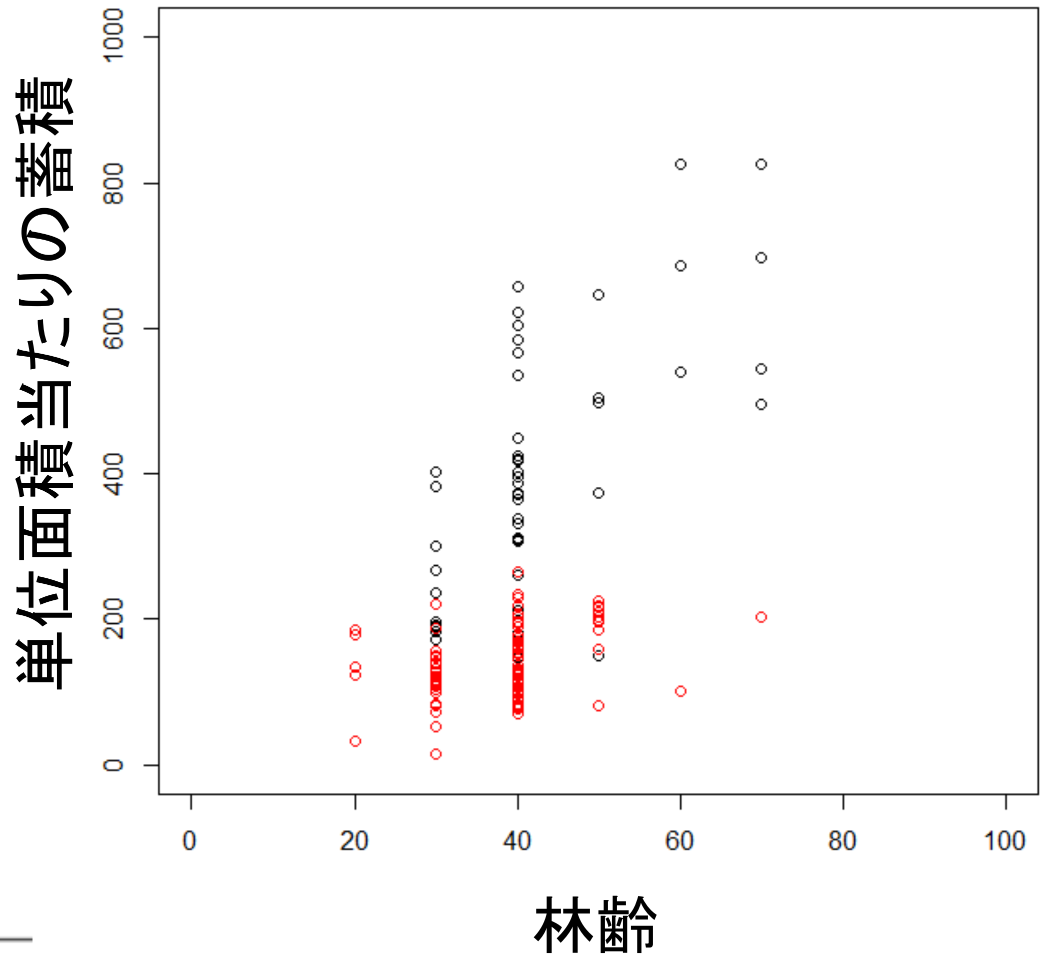
$$c_{i,j} = PNV_{i,j}$$

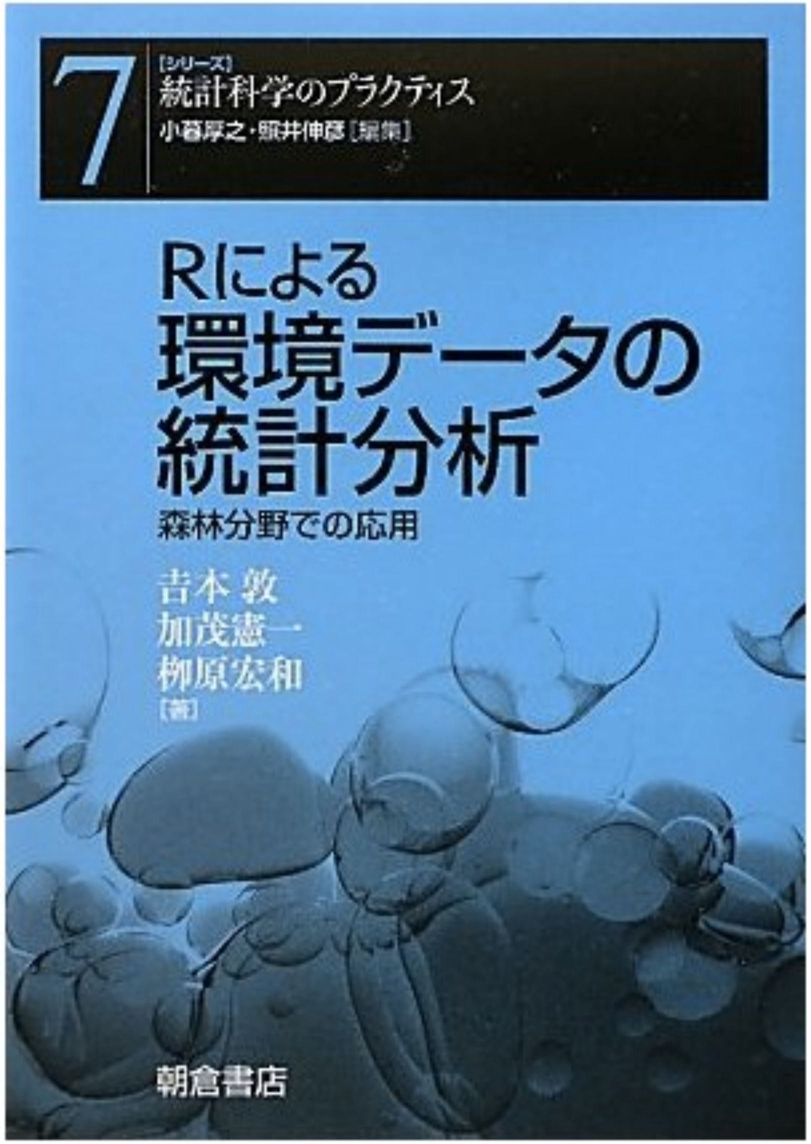


広域森林のデータ(森林簿)

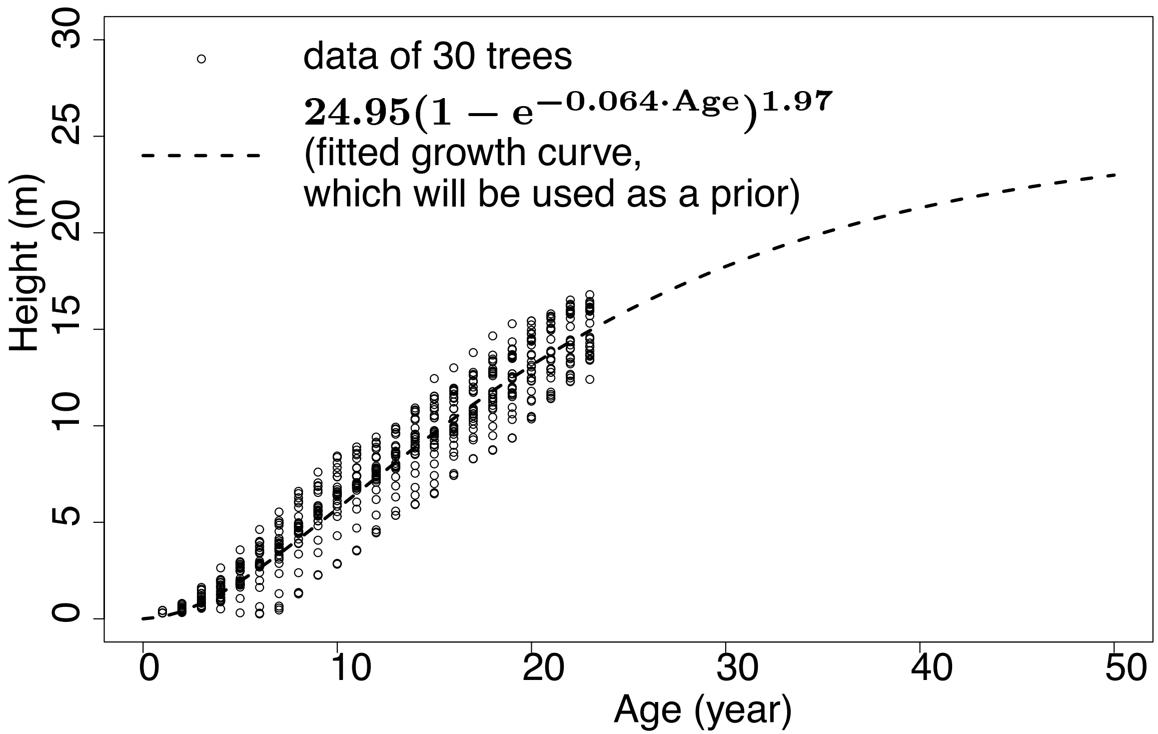
1. 樹種
2. ID
3. 林齢
4. 林分の場所
5. 所有者
6. 面積
7. 蓄積量

$$V = \frac{\text{蓄積量}}{\text{面積}} (\mathrm{m}^3/\mathrm{ha})$$

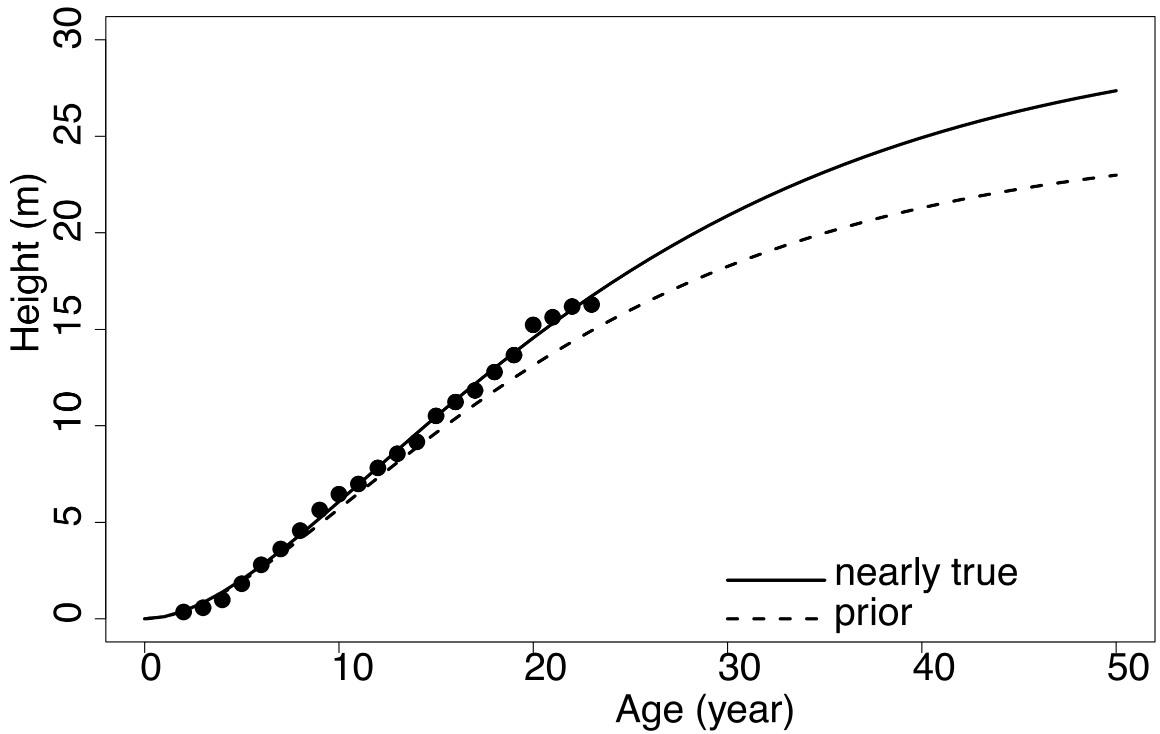



7
[シリーズ]
統計科学のプラクティス
小暮厚之・照井伸彦［編集］
Rによる
環境データの
統計分析
森林分野での応用
吉本 敦
加茂憲一
柳原宏和
［著］
朝倉書店

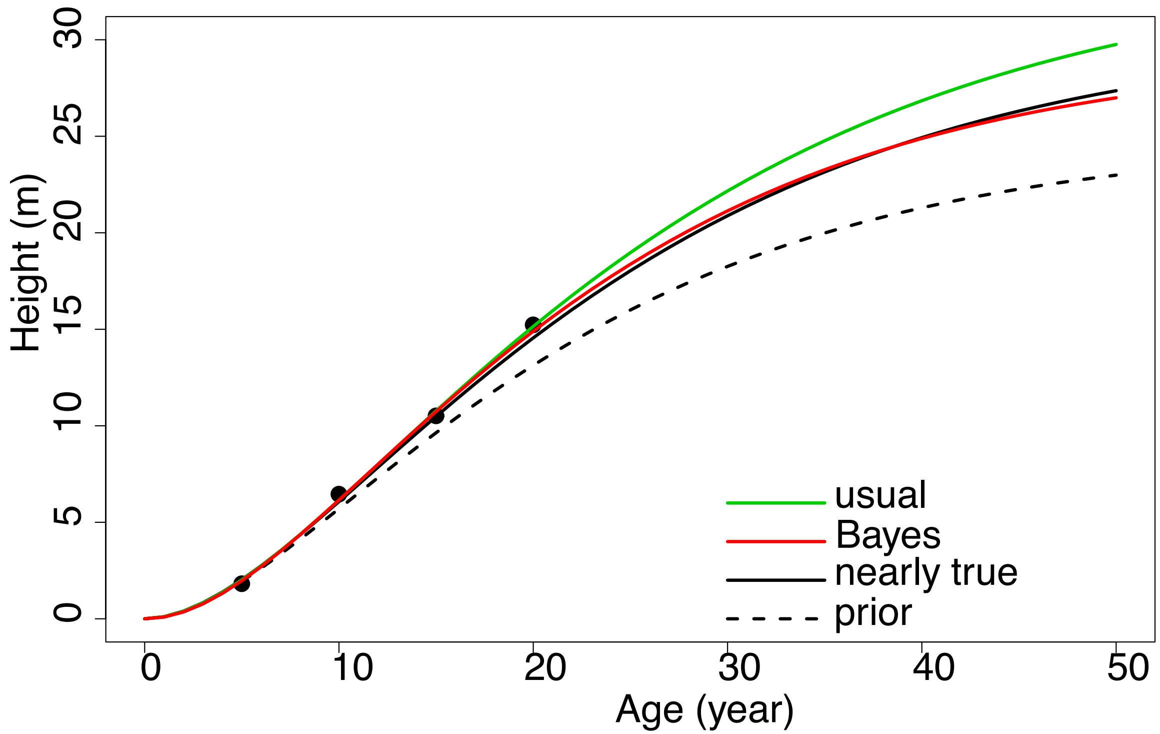
スギ30本の樹高データ



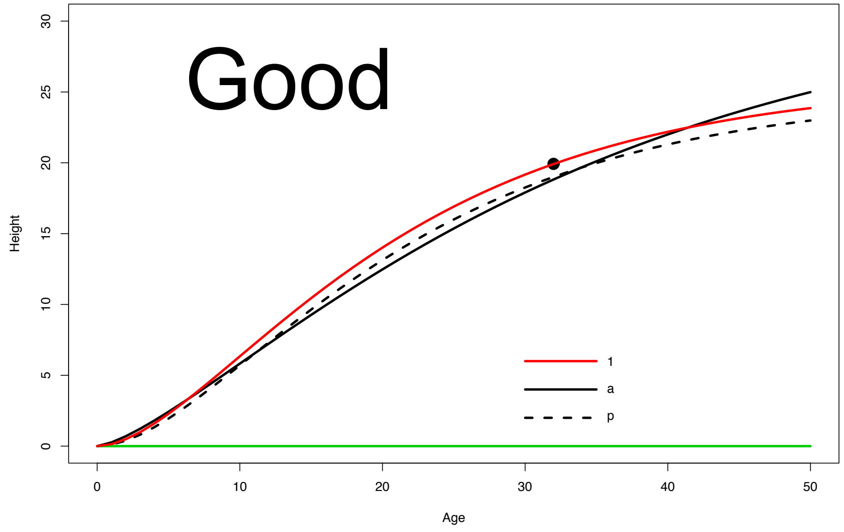
全てのデータがなくても成長曲線は推定できるのか？

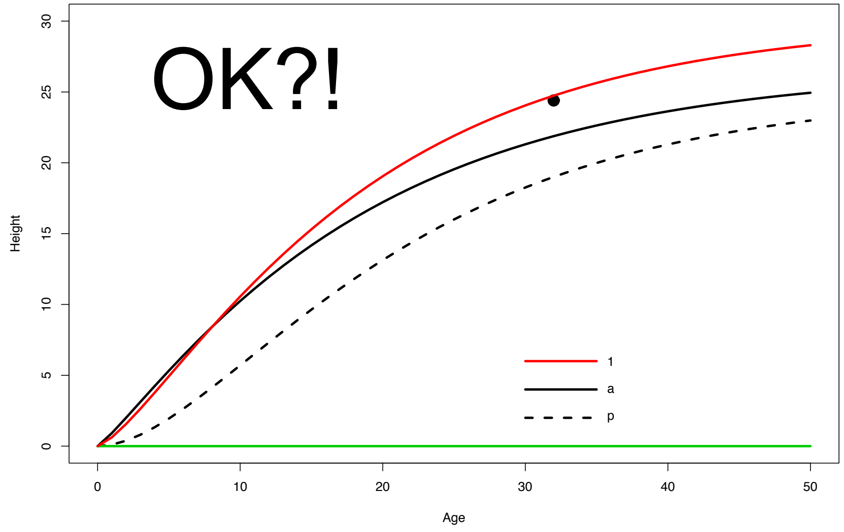


事前情報を使用したベイズ推定



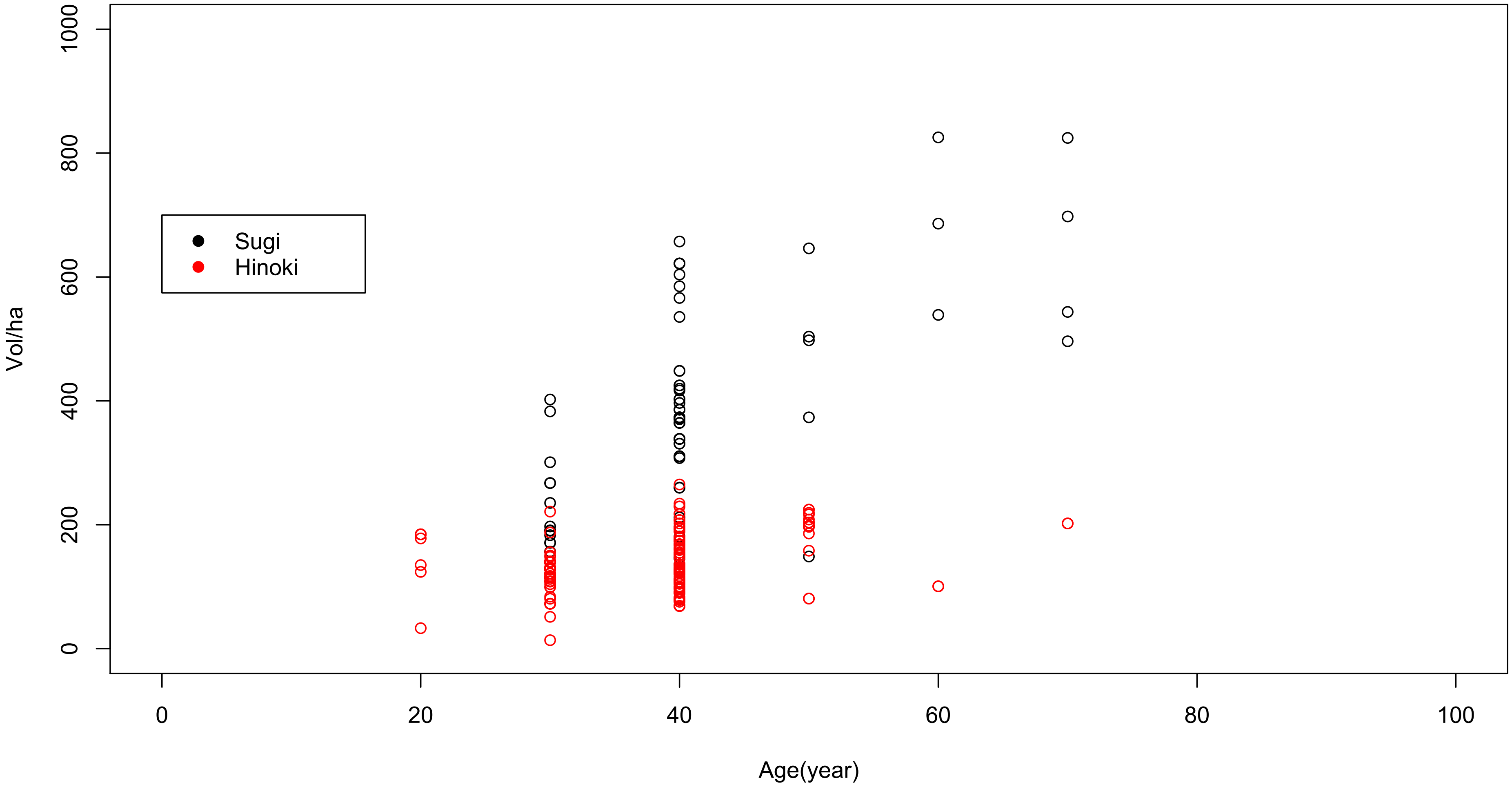
１点のデータではどうか？



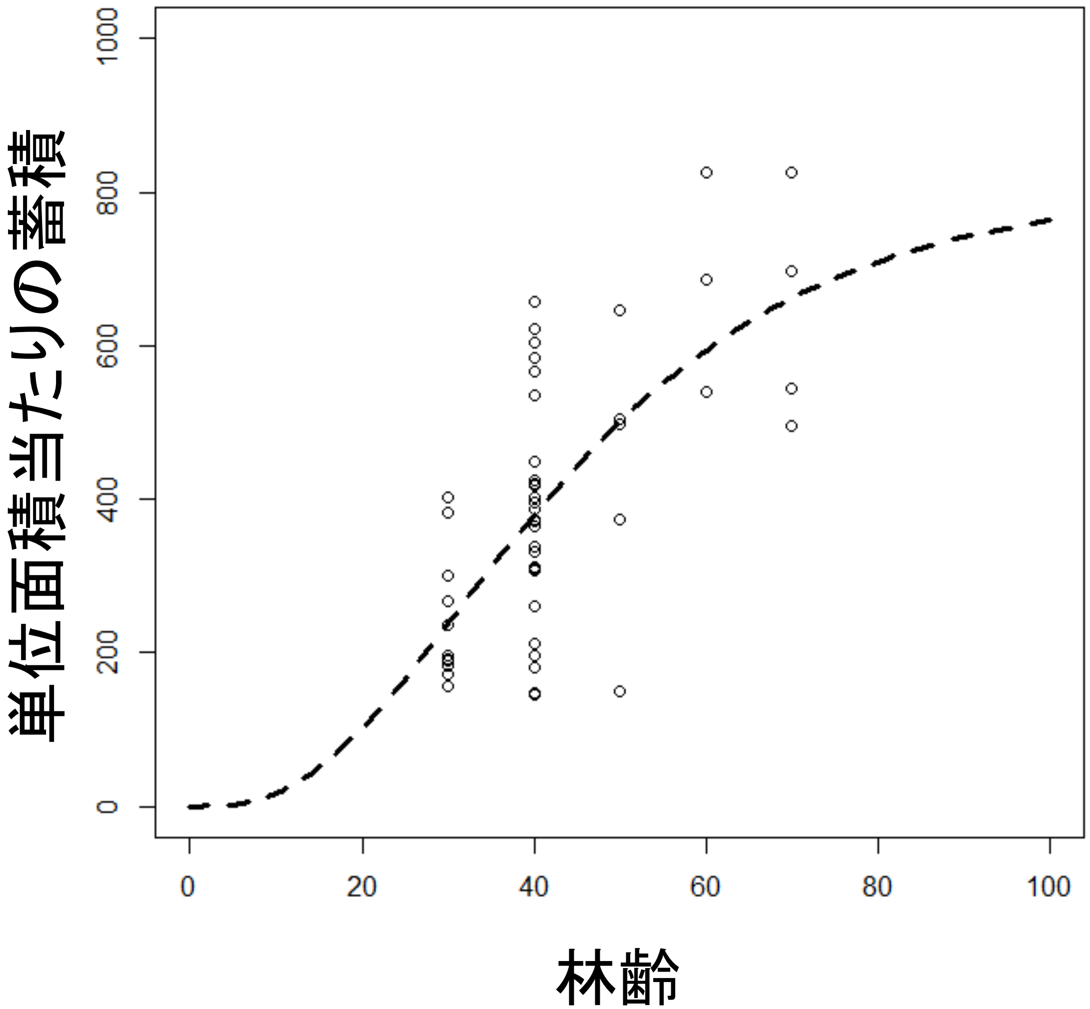


事前情報のアップデートによるRecursive Approach

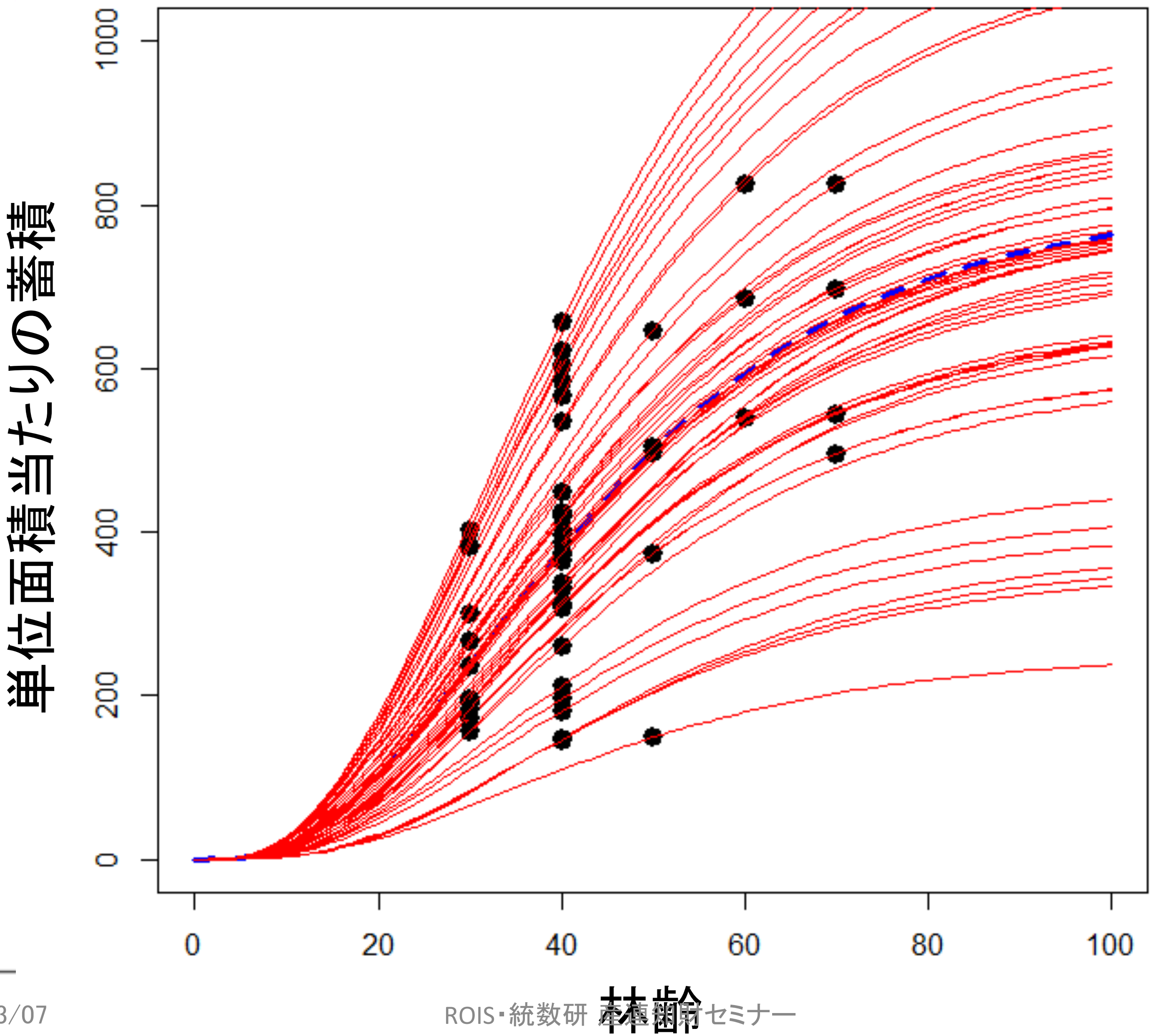
樹種により分類してベイズ推定



事前情報からの成長曲線の推定:ベイズ推定



事前情報から最もらしい予測



成長予測 + 最適化による計画

＋

事前情報 + なんらかのデータ

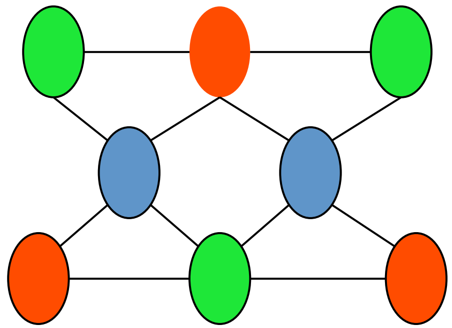
＝

許容できる最適解の探求

統計手法を駆使すれば
森林簿、収穫表などの既存のデータで
予測モデルの構築は可能！
それを用いて最適森林管理モデルの構築は可能！

- 炭素吸収量の推定
- バイオマス発電の導入
- 生物多様性への対応
- 林業従事者の育成計画
- サプライチェーンの構築
- などなど

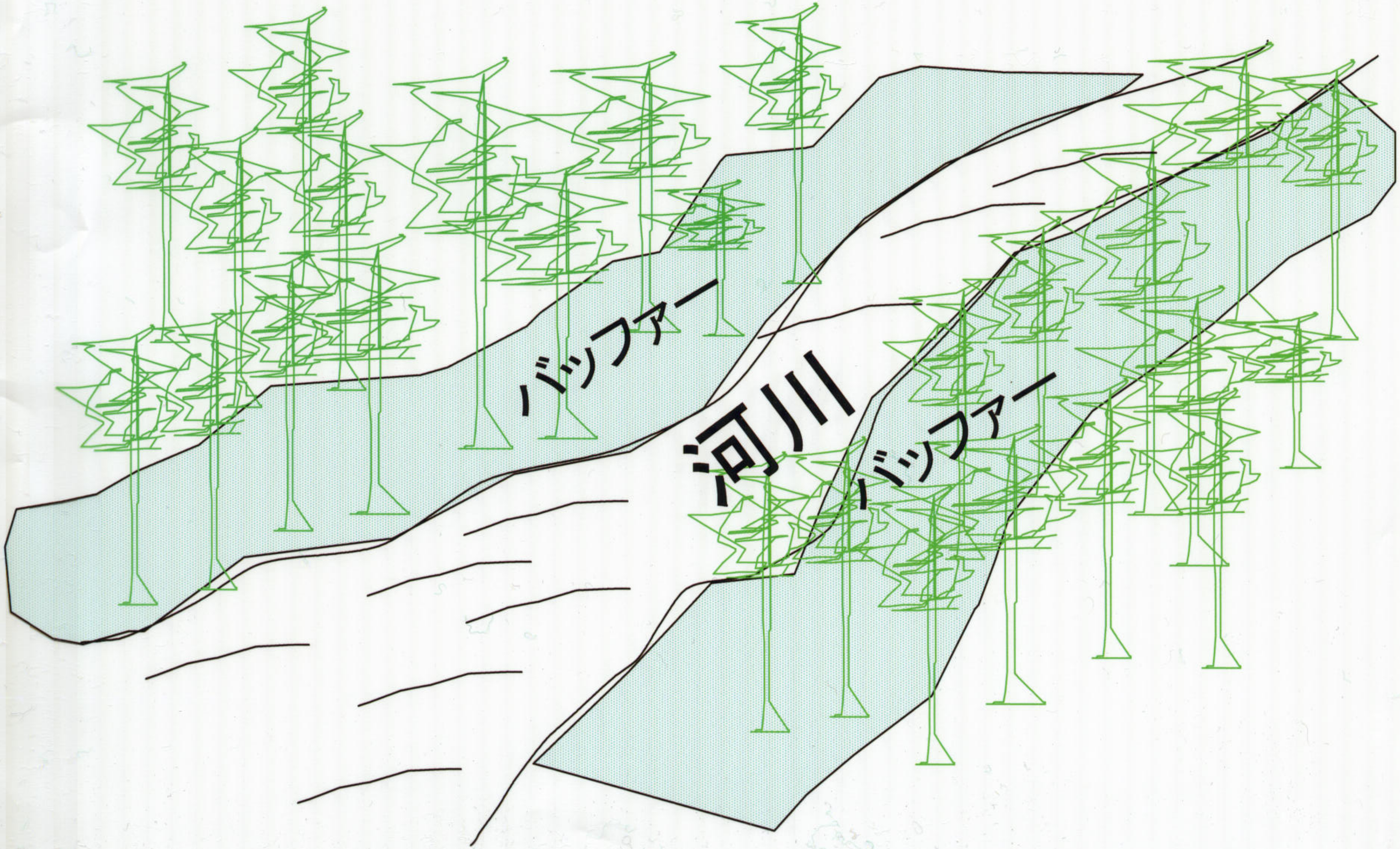
森林ランドスケープと環境の時代



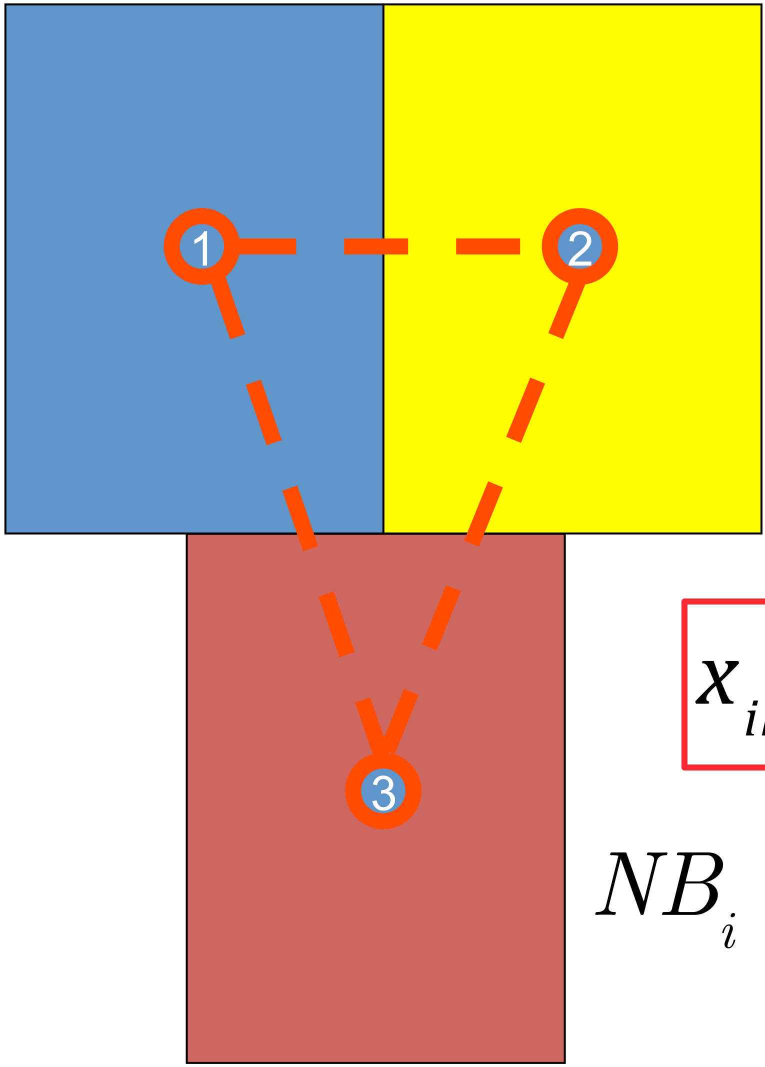
隣接林分での同時施業の回避

例: 林縁効果
グリーン・アップ制約

陸水生態系維持のための流域管理



隣接関係



$$x_{ih} = \begin{cases} 1 & \text{i林分へh施業を施す場合} \\ 0 & \text{その他} \end{cases}$$

$$x_{ih} + x_{jk} \le 1, \quad \forall i, j \in NB_i$$

NB_i : *i*林分に隣接する林分群

*h*と*k*施業は同時期伐採

隣接制約による定式化

Max PNV

Model I（基本構造）

$$\max Z = \sum_{i=1}^{m} \sum_{h=1}^{H} c_{ih} \times x_{ih}$$

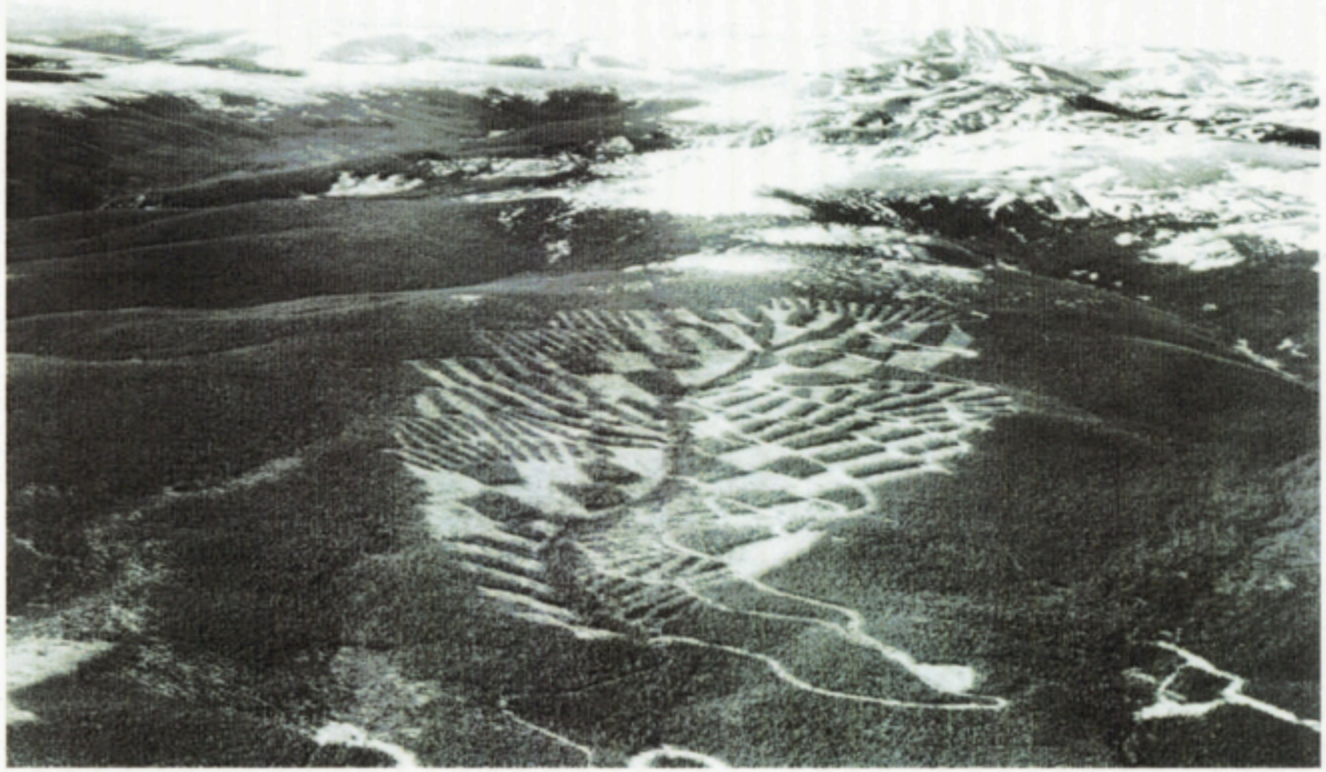
$st.$

$$\sum_{h=0}^{H} x_{ih} = 1, \quad \forall i$$ 土地利用制約

$$(1-\alpha)v_0 \le \sum_{i=1}^{m} \sum_{h=1}^{H} v_{ih}^{(p)} \times x_{ih} \le (1+\alpha)v_0, \quad p = 1, 2, \ldots, T$$ 生産量制約

$$x_{ih} + x_{jk} \le 1, \quad \forall i, j \in NB_i$$ hとk施業は同時期伐採

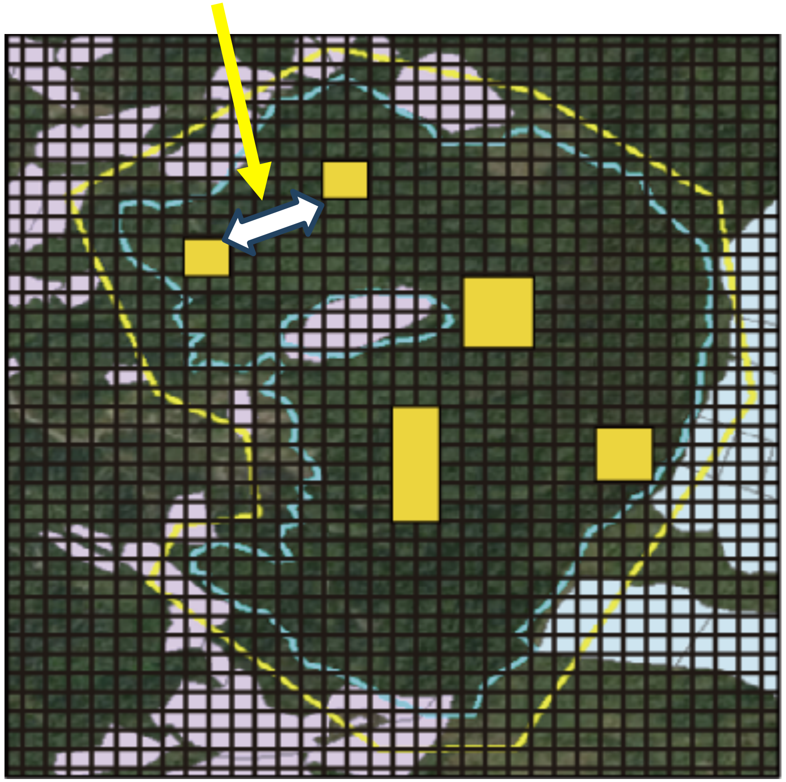
隣接制約



Wildlife Management
Animal Corridor
Landscape Management

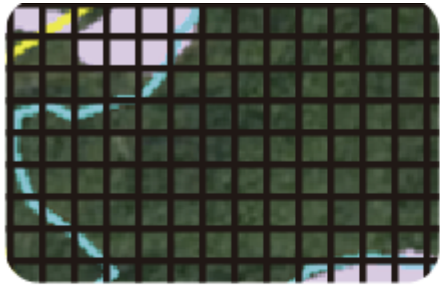
収穫に対する空間的制約？

Green-belt Constraints
Coral-belt？



Grid Segmentation

Grid Aggregation



Green-up Constraints
Coral-up？

Within-Distance Adjacency

9 x 9 grid map

Within Distance of 1.5 grid size

5.66	5	4.47	4.12	4	4.12	4.47	5	5.66
5	4.24	3.61	3.16	3	3.16	3.61	4.24	5
4.47	3.61	2.83	2.24	2	2.24	2.83	3.61	4.47
4.12	3.16	2.24	1.41	1	1.41	2.24	3.16	4.12
4	3	2	1	0	1	2	3	4
4.12	3.16	2.24	1.41	1	1.41	2.24	3.16	4.12
4.47	3.61	2.83	2.24	2	2.24	2.83	3.61	4.47
5	4.24	3.61	3.16	3	3.16	3.61	4.24	5
5.66	5	4.47	4.12	4	4.12	4.47	5	5.66

5.66	5	4.47	4.12	4	4.12	4.47	5	5.66
5	4.24	3.61	3.16	3	3.16	3.61	4.24	5
4.47	3.61	2.83	2.24	2	2.24	2.83	3.61	4.47
4.12	3.16	2.24	1.41	1	1.41	2.24	3.16	4.12
4	3	2	1	0	1	2	3	4
4.12	3.16	2.24	1.41	1	1.41	2.24	3.16	4.12
4.47	3.61	2.83	2.24	2	2.24	2.83	3.61	4.47
5	4.24	3.61	3.16	3	3.16	3.61	4.24	5
5.66	5	4.47	4.12	4	4.12	4.47	5	5.66

Within-Distance Adjacency

9 x 9 grid map

Within Distance of 2.5 grid size

5.66	5	4.47	4.12	4	4.12	4.47	5	5.66
5	4.24	3.61	3.16	3	3.16	3.61	4.24	5
4.47	3.61	2.83	2.24	2	2.24	2.83	3.61	4.47
4.12	3.16	2.24	1.41	1	1.41	2.24	3.16	4.12
4	3	2	1	0	1	2	3	4
4.12	3.16	2.24	1.41	1	1.41	2.24	3.16	4.12
4.47	3.61	2.83	2.24	2	2.24	2.83	3.61	4.47
5	4.24	3.61	3.16	3	3.16	3.61	4.24	5
5.66	5	4.47	4.12	4	4.12	4.47	5	5.66

5.66	5	4.47	4.12	4	4.12	4.47	5	5.66
5	4.24	3.61	3.16	3	3.16	3.61	4.24	5
4.47	3.61	2.83	2.24	2	2.24	2.83	3.61	4.47
4.12	3.16	2.24	1.41	1	1.41	2.24	3.16	4.12
4	3	2	1	0	1	2	3	4
4.12	3.16	2.24	1.41	1	1.41	2.24	3.16	4.12
4.47	3.61	2.83	2.24	2	2.24	2.83	3.61	4.47
5	4.24	3.61	3.16	3	3.16	3.61	4.24	5
5.66	5	4.47	4.12	4	4.12	4.47	5	5.66

Within-Distance Adjacency

9 x 9 grid map
Within Distance of 3.5 grid size

5.66	5	4.47	4.12	4	4.12	4.47	5	5.66
5	4.24	3.61	3.16	3	3.16	3.61	4.24	5
4.47	3.61	2.83	2.24	2	2.24	2.83	3.61	4.47
4.12	3.16	2.24	1.41	1	1.41	2.24	3.16	4.12
4	3	2	1	0	1	2	3	4
4.12	3.16	2.24	1.41	1	1.41	2.24	3.16	4.12
4.47	3.61	2.83	2.24	2	2.24	2.83	3.61	4.47
5	4.24	3.61	3.16	3	3.16	3.61	4.24	5
5.66	5	4.47	4.12	4	4.12	4.47	5	5.66

5.66	5	4.47	4.12	4	4.12	4.47	5	5.66
5	4.24	3.61	3.16	3	3.16	3.61	4.24	5
4.47	3.61	2.83	2.24	2	2.24	2.83	3.61	4.47
4.12	3.16	2.24	1.41	1	1.41	2.24	3.16	4.12
4	3	2	1	0	1	2	3	4
4.12	3.16	2.24	1.41	1	1.41	2.24	3.16	4.12
4.47	3.61	2.83	2.24	2	2.24	2.83	3.61	4.47
5	4.24	3.61	3.16	3	3.16	3.61	4.24	5
5.66	5	4.47	4.12	4	4.12	4.47	5	5.66

Within-Distance Adjacency among individual grid units

Normal Adjacency

0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0
0	0	0	1	0	1	0	0	0
0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

Within-Distance Adjacency

0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	1	1	1	0	0	0
0	0	0	1	0	1	0	0	0
0	0	0	1	1	1	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

$$a^{v}_{hl} \times x_{ih} + a^{v}_{lh} \times x_{jl} \leq 1, \ \forall j \in B^{d}_{i}, \ \forall i, t = 1, \ldots, T$$

Within-Distance Adjacency among individual grid units

Normal Adjacency

0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0
0	0	0	1	0	1	0	0	0
0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

Within-Distance Adjacency

0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	1	1	1	0	0	0
0	0	1	1	1	1	1	0	0
0	0	1	1	0	1	1	0	0
0	0	1	1	1	1	1	0	0
0	0	0	1	1	1	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

$$a^{v}_{hl}\,\bar{x}_{ih} + a^{v}_{lh}\,\bar{x}_{jl} \le 1, \ \forall j \in B^{d}_{i}, \forall i, t = 1, \ldots, T$$

Within-Distance Adjacency among individual grid units

Normal Adjacency

0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0
0	0	0	1	0	1	0	0	0
0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

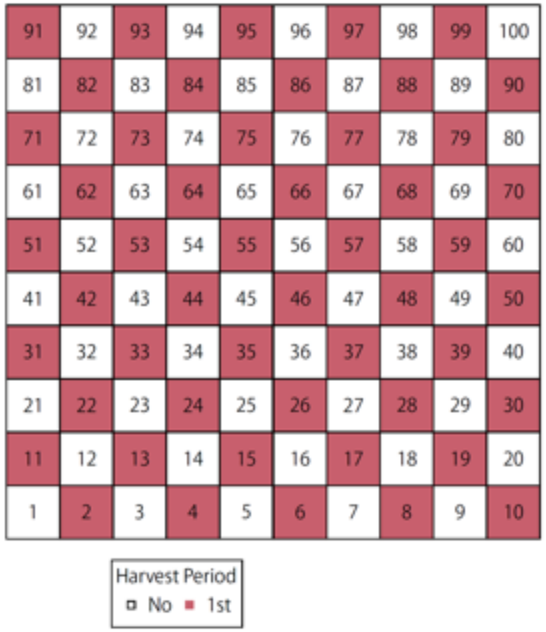
Within-Distance Adjacency

0	0	0	0	0	0	0	0	0
0	0	0	1	1	1	0	0	0
0	0	1	1	1	1	1	0	0
0	1	1	1	1	1	1	1	0
0	1	1	1	0	1	1	1	0
0	1	1	1	1	1	1	1	0
0	0	1	1	1	1	1	0	0
0	0	0	1	1	1	0	0	0
0	0	0	0	0	0	0	0	0

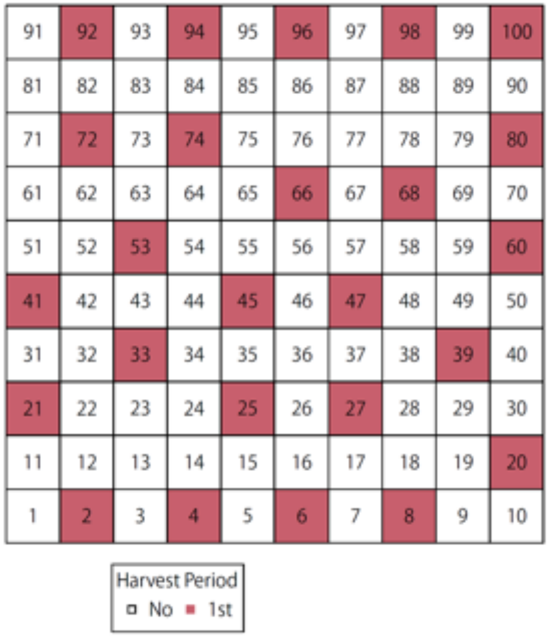
$$a^v_{hl} \cdot x_{ih} + a^v_{lh} \cdot x_{jl} \le 1, \ \forall j \in B_i^d, \forall i, t = 1, \ldots, T$$

グリーンベルト幅の違いによる比較

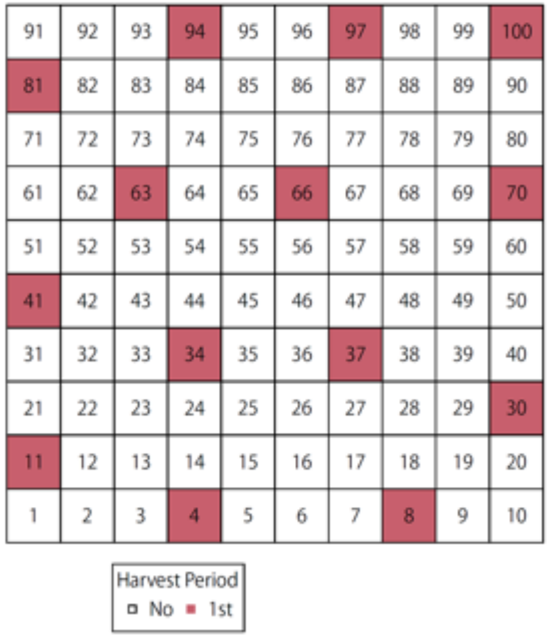
ex100gd_0AGG_0WDAC_1P : No Harvests: 50 units



ex100gd_0AGG_15WDAC_1P : No Harvests: 75 units



ex100gd_0AGG_25WDAC_1P : No Harvests: 86 units



サンゴの資源管理？